

Behavioral Economics

Topic 2: Intertemporal Choice
Lecture 10: The Standard Model
Exponential Discounting

Professor Bushong



Taking Stock



Tuesday: Interest rates, compounding, and present discounted value

Today: The standard model

- Exponential discounting, consumption and saving, and time consistency
- Measuring discount functions and testing the model against the evidence

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Logistics:

- Problem Set 3 is due before class on Thursday, October 22
- Tuesday, October 13: an alternative model — present bias

Discounting of Utility



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 - This model is as if we have some sort of "interest rate" over streams of utility that is analogous to interest rates over money

Exponential Discounting



Exponential discounting: When a person receives utility at different points in time, she seeks to maximize her *intertemporal utility*:

$$\begin{aligned} U &\equiv u_1 + \delta u_2 + \delta^2 u_3 + \dots + \delta^{T-1} u_T \\ &= \sum_{t=1}^T \delta^{t-1} u_t. \end{aligned}$$

- u_t is her *instantaneous utility* in period t (“well-being” in period t).
- δ is her *discount factor*.

Discount Factors and Discount Rates



Instead of using

$$U = \sum_{t=1}^T \delta^{t-1} u_t$$

we sometimes use

$$U \equiv \sum_{t=1}^T \frac{1}{(1 + \rho)^{t-1}} u_t.$$

- ρ is the person's *discount rate* (rate of time preference).
- Reminder: δ is the person's *discount factor*.
- Note: $\delta = 1/(1 + \rho)$
- The market interest rate r prices saving and borrowing; ρ describes personal time preference.

(A) 90 utils in period 2 vs. (B) 160 utils in period 4

Which do you prefer?

[illegible]

Example 2: Suppose you have the opportunity to give up 5 utils now in order to gain 2 utils for each of the next three periods. Do you take it?

Two-Period Consumption and Saving



Example 3: Two-Period Saving-Consumption Decisions

Let c_1 be your consumption expenditures in period 1, and let c_2 be your consumption expenditures in period 2. Hence, in the end you must choose a consumption bundle (c_1, c_2) .

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$$U(c_1, c_2) = u(c_1) + \delta u(c_2).$$

- Note: $u(c_t)$ is your period- t instantaneous utility as a function of your period- t consumption — typically assume $u' > 0$ and $u'' < 0$.

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Let Y_1 be your income received in period 1, and let Y_2 be your income received in period 2.

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Let Y_1 be your income received in period 1, and let Y_2 be your income received in period 2.

Let r be the market interest rate, at which you can either save or borrow.

The Two-Period Budget Constraint



What is your budget constraint?

Suppose $Y_1 > 0$ and $Y_2 = 0$

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Suppose $Y_1 > 0$ and $Y_2 = 0$ ("how-to-eat-a-cake" problem):

- Budget constraint is

$$c_1 + \frac{c_2}{1+r} \leq Y_1$$

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Suppose instead $Y_1 > 0$ and $Y_2 > 0$.

- Budget constraint is

$$c_1 + \frac{c_2}{1+r} \leq Y_1 + \frac{Y_2}{1+r}.$$

Present Value and the Budget Constraint



General Principle: If there are no liquidity constraints — that is, if you can borrow and save at the same market interest rate — then your budget constraint will take the form:

$$PDV \text{ of Consumption Expenditures} \leq PDV \text{ of Income Flows.}$$

Optimal Consumption with Log Utility



Choose (c_1, c_2) to maximize $u(c_1) + \delta u(c_2)$ subject to

$$c_1 + \frac{c_2}{1+r} \leq Y_1 + \frac{Y_2}{1+r} \equiv W.$$

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At an interior optimum, the marginal utility lost by saving a dollar today matches the discounted marginal utility gained tomorrow:

$$u'(c_1) = \delta(1+r)u'(c_2).$$

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For $u(c) = \ln c$, this gives $c_2 = \delta(1+r)c_1$. Using the budget constraint,

$$c_1 = \frac{W}{1+\delta} \quad \text{and} \quad c_2 = \frac{\delta(1+r)W}{1+\delta}.$$

Practice: Two-Period Optimization



Consider a simple two-period model of intertemporal choice. Suppose that Rocky receives income Y_1 in period 1 and additional income Y_2 in period 2. (He dies in his fight against Ivan Drago immediately after period 2). The market interest rate at which Rocky can both borrow and save is 4%. Finally, the person's preferences are given by

$$U(c_1, c_2) = \frac{3}{2} (c_1)^{2/3} + \delta \frac{3}{2} (c_2)^{2/3}.$$

- (a) Derive the budget constraint that the person faces.
- (b) Solve for the optimal c_1 and c_2 as a function of δ .
- (c) When is the person saving in period 1? When is the person borrowing in period 1?

Borrowing and Saving at Different Rates



What if there are liquidity constraints?

Consider Example 3, except suppose that when you borrow, you must pay an interest rate $r_B > r$.

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■ Budget constraint is:

$$c_1 + \frac{c_2}{1+r} \leq Y_1 + \frac{Y_2}{1+r} \equiv W_S \quad \text{if } c_1 \leq Y_1$$

$$c_1 + \frac{c_2}{1+r_B} \leq Y_1 + \frac{Y_2}{1+r_B} \equiv W_B \quad \text{if } c_1 > Y_1$$

Saving, Borrowing, or Neither?



At the endowment (Y_1, Y_2) , compare the marginal utility of a dollar today with its discounted value tomorrow. Let $r_B > r$.

- **Save** if $u'(Y_1) < \delta(1 + r)u'(Y_2)$.

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- **Borrow** if $u'(Y_1) > \delta(1 + r_B)u'(Y_2)$.
- **Neither** if

$$\delta(1 + r)u'(Y_2) \leq u'(Y_1) \leq \delta(1 + r_B)u'(Y_2).$$

The borrowing–saving wedge can make $(c_1, c_2) = (Y_1, Y_2)$ optimal.

Consumption over Many Periods



Example 4: T -Period Saving-Consumption Decisions

Suppose that you consume in T different periods, and let c_t denote your consumption expenditures in period t . Hence, in the end you must choose a consumption bundle $(c_1, c_2, \dots, c_T)$.

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Let r be the market interest rate, and assume no liquidity constraints.

- Implication: You will choose $(c_1, c_2, \dots, c_T)$ to maximize $U(c_1, c_2, \dots, c_T)$ subject to

$$c_1 + \frac{c_2}{1+r} + \dots + \frac{c_T}{(1+r)^{T-1}} \leq Y_1 + \frac{Y_2}{1+r} + \dots + \frac{Y_T}{(1+r)^{T-1}}.$$

Lifetime Wealth and Log Utility



Example 4 with $u(c) = \ln c$:

- Goal: Choose $(c_1, c_2, \dots, c_T)$ to maximize

$$U(c_1, c_2, \dots, c_T) = \ln c_1 + \delta \ln c_2 + \dots + \delta^{T-1} \ln c_T$$

subject to

$$c_1 + \frac{c_2}{1+r} + \dots + \frac{c_T}{(1+r)^{T-1}} \leq W$$

where

$$W \equiv Y_1 + \frac{Y_2}{1+r} + \dots + \frac{Y_T}{(1+r)^{T-1}}.$$

Consumption Growth and Smoothing



Relationships between variables:

- For each $t > 1$:

$$c_t = [\delta(1+r)]^{t-1} c_1$$

or
$$c_t = \delta(1+r)c_{t-1}.$$

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Solving for consumption:

$$c_1 = \frac{W}{1 + \delta + \dots + \delta^{T-1}} \quad \text{and} \quad c_t = \frac{[\delta(1+r)]^{t-1} W}{1 + \delta + \dots + \delta^{T-1}}.$$

General Version of Discounted Utility



Now consider a more general version of discounted-utility model:

$$U^t = \sum_{x=0}^{T-t} D(x) u_{t+x}.$$

- U^t is intertemporal utility from perspective of period t .

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- Exponential discounting **assumes** $D(x) = \delta^x$.

Impatience and Constant Discounting



(1) Impatience: For $\delta < 1$, $D(x)$ is monotonically declining in x .

Impatience and Constant Discounting



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- This represents an evenhandedness in how you view time.
- If we're thinking in terms of years, how you feel about this year vs. next year is the same as how you feel about next year vs. the following year is the same as how you feel about 5 years from now vs. 6 years from now.

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- If we're thinking in terms of days, how you feel about today vs. tomorrow is the same as how you feel about tomorrow vs. the next day is the same as how you feel about 100 days from now vs. 101 days from now.

Time Consistency



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- Your relative preference between two calendar dates is independent of when you are asked (independent of your perspective).
- Let's work through an example of how relative preferences depend on one's perspective under exponential discounting....

Relative Weights as Time Passes



From a period-1 perspective, where your intertemporal preferences are

$$U^1 = \sum_{x=0}^{T-1} D(x) u_{1+x},$$

How do you weight:

- period 2 vs. period 3?

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- period 3 vs. period 5?

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From a period-2 perspective, where your intertemporal preferences are

$$U^2 = \sum_{x=0}^{T-2} D(x)u_{2+x},$$

How do you weight: Period 2 vs. period 3?

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How do you weight:

- period 2 vs. period 3?
- period 3 vs. period 5?

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How do you weight: Period 2 vs. period 3? Period 3 vs. period 5?

Time Consistency: A Payoff Example



Example 5: Suppose you have linear instantaneous utility, and that you must choose between the following two options:

- Option A: Receive payoff V_L in period 2.
- Option B: Receive payoff V_H in period 3.

In period 1, prefer Option B when ?

In period 2, prefer Option B when ?

Evidence on Discounting

How to Measure Discount Functions?



Typical procedure elicits indifference points of the form:

$$(A \text{ at date } x) \sim (B \text{ at date } x + y)$$

How to Measure Discount Functions?



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To interpret, typically make four assumptions:

■ “The Usual Assumptions”

- (1) People obey discounted utility model.
- (2) People treat these amounts as “bursts” of consumption.
- (3) “Utility” is linear in the amount.
- (4) Normalize $D(0) = 1$.

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An implication of the usual assumptions:

$$(A \text{ now}) \sim (B \text{ at date } x)$$

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$$\iff D(0)u(A) = D(x)u(B)$$

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$$\iff \frac{D(x + y)}{D(x)} = \frac{A}{B}$$

How to Measure Discount Functions?



An alternative procedure elicits WTP now for something to be received later — e.g., if WTP up to A now to obtain B at date x , this implies:

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An alternative procedure elicits WTP now for something to be received later — e.g., if WTP up to A now to obtain B at date x , this implies:

$$(-A \text{ now \& } +B \text{ at date } x) \sim (\text{no changes})$$

Applying the usual assumptions here yields:

$$D(0)u(-A) + D(x)u(B) = 0$$

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$$\iff D(x)B = A$$

$$\iff D(x) = \frac{A}{B}$$

Some Evidence from Thaler (1981)



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- c. (\$15 now) $\sim$ (\$100 in 3 years)

Implications under the usual assumptions:

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Implications under the usual assumptions:

- a. $D(3 \text{ months}) = \frac{15}{30} = 0.50$.

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- a. $D(3 \text{ months}) = \frac{15}{30} = 0.50$.
- b. $D(1 \text{ year}) = \frac{15}{60} = 0.25$.

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- a. $D(3 \text{ months}) = \frac{15}{30} = 0.50$.
- b. $D(1 \text{ year}) = \frac{15}{60} = 0.25$.
- c. $D(3 \text{ years}) = \frac{15}{100} = 0.15$.

Some Evidence from Thaler (1981)



We can convert each $D(x)$ into an implicit yearly discount rate:

Definition: The *average continuously compounded yearly discount rate applied to delay x* (where x is specified in years) is the ρ_c such that

$$e^{-\rho_c x} = D(x) \quad \text{or} \quad \rho_c = \frac{1}{x}(-\ln D(x)).$$

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- c. $D(3 \text{ years}) = 0.15 \implies 63\%$ yearly discounting.

More Evidence from Thaler (1981)



Thaler (1981) also found that people were indifferent between:

- e. (\$250 now) $\sim$ (\$300 in 3 months)
- f. (\$250 now) $\sim$ (\$350 in 1 year)
- g. (\$250 now) $\sim$ (\$500 in 3 years)

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Implications under the usual assumptions:

■ e. $D(3 \text{ months}) = \frac{250}{300} = 0.83 \implies 73\%$ yearly discounting.

■ f. $D(1 \text{ year}) = \frac{250}{350} = 0.71 \implies 34\%$ yearly discounting.

■ g. $D(3 \text{ years}) = \frac{250}{500} = 0.50 \implies 23\%$ yearly discounting.

Conclusions from Thaler (1981)



Conclusion #1: The amount matters — there is more discounting for smaller amounts (“magnitude effect”).

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Conclusions from Thaler (1981)



Conclusion #1: The amount matters — there is more discounting for smaller amounts (“magnitude effect”).

But the key conclusion is:

Conclusion #2: For either amount, discount rates are higher in the short run than in the long run (sometimes referred to as “declining discount rates”).

- This finding is inconsistent with exponential discounting!

A Hypothetical Preference Reversal



Consider the following hypothetical choice scenarios:

Choice 1:

[10 M&M's now] vs. [15 M&M's tomorrow]

A Hypothetical Preference Reversal



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Choice 1:

[10 M&M's now] vs. [15 M&M's tomorrow]

Choice 2:

[10 M&M's in 7 days] vs. [15 M&M's in 8 days]

A Hypothetical Preference Reversal



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Choice 1:

[10 M&M's now] vs. [15 M&M's tomorrow]

Choice 2:

[10 M&M's in 7 days] vs. [15 M&M's in 8 days]

A plausible pattern:

(10 M&M's now) $\succ$ (15 M&M's tomorrow)

(10 M&M's in 7 days) $\prec$ (15 M&M's in 8 days)

Preference Reversals: The Implication



Implication: Under the usual assumptions, this pattern implies

$$D(0)10 > D(1)15 \iff \frac{D(1)}{D(0)} < \frac{2}{3}$$

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Hence: $\frac{D(1)}{D(0)} < \frac{D(8)}{D(7)}$

Conclusion: If you exhibit this pattern, then you are less patient toward the now-vs.-near-future tradeoff than you are toward the near-future-vs.-further-future tradeoff.

Preference Reversals in the Lab



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For instance, for each of these 23 subjects, they find an $x > 0$ and a $y > 0$ such that the subject's preferences are

$$(\$45 \text{ now}) \succ (\$52 \text{ in } y \text{ days})$$

$$(\$45 \text{ in } x \text{ days}) \prec (\$52 \text{ in } x + y \text{ days})$$

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Much as above, under the usual assumptions, this implies

$$\frac{D(y)}{D(0)} < \frac{D(x+y)}{D(x)}$$

$\Rightarrow$ Again, less patient in choices involving the present vs those involving only the future.

Evidence of “Hyperbolic Discounting”



Another approach to the same type of data is to directly compare two functional forms:

- Exponential Discounting: $D(x) = e^{-kx}$
- Hyperbolic Discounting: $D(x) = \frac{1}{1+kx}$

These comparisons test whether discounting is constant or instead becomes less steep as the delay increases. An example follows.

Evidence of “Hyperbolic Discounting”



For instance, Kirby (1997) elicited WTP's for \$20 to be received in x days, where each subject answered for every odd x between 1 and 29.

He then tested for each subject whether their discount function was better fit by the exponential functional form or the hyperbolic functional form.

Evidence of “Hyperbolic Discounting”



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He then tested for each subject whether their discount function was better fit by the exponential functional form or the hyperbolic functional form.

Results: Hyperbolic was a better fit for 59 of 67 subjects, exponential was a better fit for 6 subjects, and for 2 subjects the functions were equally good.

Conclusions from the Evidence



The evidence we have considered points to a common pattern:

- Discounting is more extreme in the short run than in the long run (sometimes referred to as “declining discount rates”).

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Note: This suggests a time inconsistency of an impulsive nature:

- When thinking about some future period, you'd like to behave relatively patiently; but when the time comes to actually choose your behavior, you want to behave relatively impatiently.

Next Class



Today: The standard exponential model, its predictions, and the evidence

Tuesday, October 13: An alternative model — present bias

- The β, δ model and time-inconsistent choices
- How awareness of future self-control problems affects behavior

Remember: Problem Set 3 is due before class on Thursday, October 22