

Behavioral Economics

Topic 2: Intertemporal Choice

Lecture 9: Warm-Up on Interest Rates
and Present Discounted Value

Professor Bushong



Taking Stock



Last class: Exam 1

Today:

- A new topic: choice over time
- Interest rates, compounding, and present discounted value

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- Interest rates, compounding, and present discounted value

Logistics:

- Problem Set 3 is posted today — due before class on Thursday, October 22
- Start early; the exponential model begins next class

Overview of Topic 2



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Today's Lecture



Warm-Up: Interest Rates, Compounding, PDV,....

The standard model of choice over time begins next class

This is a great time to ask lots of questions.

Interest Rates and Compounding



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- After 3 years, you'll have $(\$1210) * (1.10) = \$1331$.

More generally:

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More generally:

- If you put $\$P$ into a bank account that pays interest rate r per year, its future value in T years will be $(\$P) * (1 + r)^T$
- Notice the “compounding”: earning interest on past interest earned

Discrete-Time Models



Suppose there is some set of periods $0, 1, 2, \dots, T$ (perhaps $T = \infty$).

- Note: The length of a period might be one year, one month, one day, or whatever is most appropriate for the particular application.

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If P_t is the amount (“principal”) in your bank account in period t , then:

- $P_1 = (1 + r) * P_0$
- $P_2 = (1 + r)^2 * P_0$

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- $P_{t+x} = (1 + r)^x * P_t$

Present Discounted Value (PDV)



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- Solve for P_0 in the equation $P_1 = (1 + r)P_0$
- Answer: \$1000 — because $(1.10) * (\$1000) = \$1100$.

Definition: Given per-period interest rate r , the *present discounted value* (or sometimes just *present value*) of $\$P$ to be paid T periods in the future is

$$\frac{P}{(1 + r)^T}$$

Present Discounted Value (PDV)



Some *PDV*'s for $P = \$1000$ and yearly compounding:

r	1 Year	2 Years	3 Years	10 Years	20 Years
3%	\$971	\$943	\$915	\$744	\$554
4%	\$962	\$925	\$889	\$676	\$456
5%	\$952	\$907	\$864	\$614	\$377
6%	\$943	\$890	\$840	\$558	\$312
7%	\$935	\$873	\$816	\$508	\$258

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■ Answer: Add up the individual *PDV*'s:

$$PDV = \frac{\$1100}{(1.10)} + \frac{\$1100}{(1.10)^2} + \frac{\$1100}{(1.10)^3} = \$2735.54.$$

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More generally: Given per-period interest rate r , a stream of future revenues $(R_1, R_2, \dots, R_N)$ — where revenue R_n is received in period n — has a present discounted value of

$$PDV = \frac{R_1}{(1+r)} + \frac{R_2}{(1+r)^2} + \dots + \frac{R_N}{(1+r)^N}.$$

Next Class



Today: Interest rates, compounding, and present discounted value

Thursday, October 8: The standard model of choice over time — exponential discounting

Remember:

- Problem Set 3 is due before class on Thursday, October 22