Behavioral Economics

Topic 1: Choice under Uncertainty

Lecture 6: Applications of Prospect Theory

Professor Bushong

MICHIGAN STATE UNIVERSITY

Taking Stock

Last class:

- Prospect Theory

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Today:

- Applications of Prospect Theory

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Last class:

- Prospect Theory

Today:

- Applications of Prospect Theory

Logistics:

- Problem Set 1 was due last class — detailed solutions are posted
  - Please read them carefully — treat this as part of the assignment
- Problem Set 2 is due Thursday, September 24 — you can do several problems already
- Exam 1 is Tuesday, September 29 (in class)

A Quick Reminder of Prospect Theory

Prospect Theory: A Quick Reminder

A person evaluates a prospect $(x_1, p_1; \ldots; x_n, p_n)$ according to:

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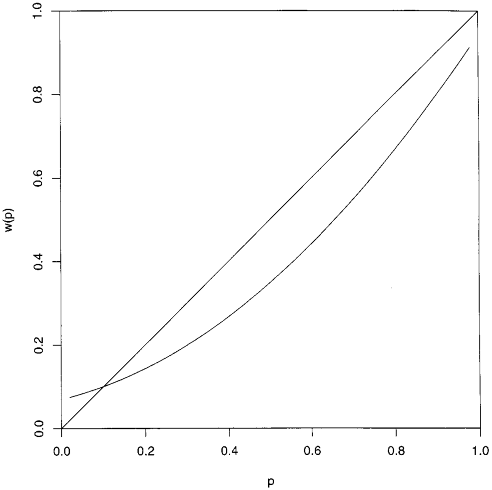
A commonly assumed form of $v(\cdot)$ is

$$v(x) = \begin{cases} x^{\alpha} & \text{if } x \geq 0 \\ \lambda(x)^{\beta} & \text{if } x \leq 0 \end{cases}$$

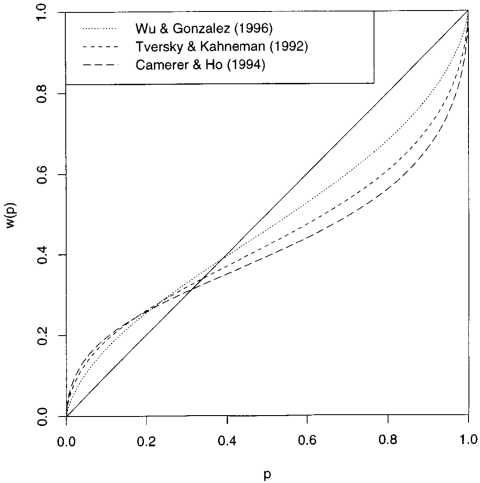
Visualizing the Probability Weighting Function

Kahneman and Tversky suggest the weighting function could look like:



Visualizing the Probability Weighting Function

Research began refining this function with focused lab evidence:



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3. Typically need more info to classify risk attitude toward lotteries with **both** gains/losses

(And you should remember these definitions!)

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- What happens if she over-estimates small probabilities?

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Applications of Prospect Theory

Our Outline

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To illustrate, we'll consider nine examples (spread across two classes)

- Only one example addresses probability weighting – but I suspect it is running around in tons of seemingly-strange behaviors.

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But **was** his colleague "irrational"?

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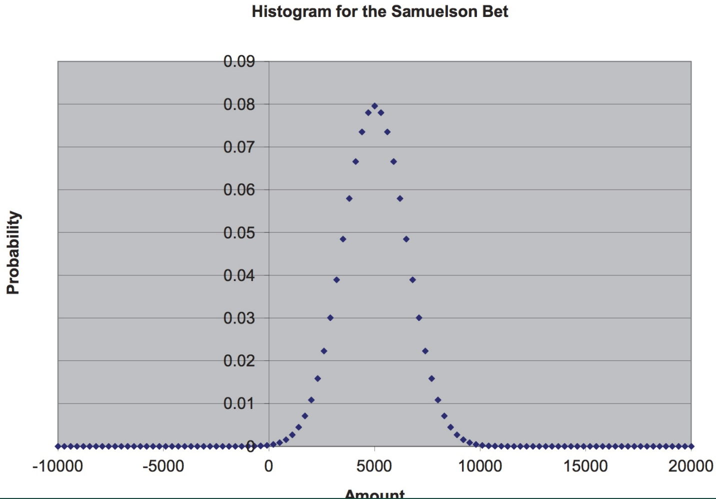

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Point: Unlike EU, loss aversion can lead a person to reject one play of the bet but to accept multiple plays of the bet.

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  - E.g., we'll do this in Applications 3-5.

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- Calibrationwise, this explanation doesn't work, because according to EU, "anything but virtual risk neutrality over modest stakes implies manifestly unrealistic risk aversion over large stakes."

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- E.g.: A 50-50 gamble of losing \$100 vs. gaining \$110.

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Rabin & Thaler's point, which should feel **very** repetitive by now:

- Calibrationwise, this explanation doesn't work, because according to EU, "anything but virtual risk neutrality over modest stakes implies manifestly unrealistic risk aversion over large stakes."
- Furthermore, loss aversion is a useful alternative.

Application #2: Risk Aversion

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With a little work, one can show that rejecting the bet implies that $\rho > 18.17026$.

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Point: The degree of risk aversion required to explain your rejection of the moderate-stakes gamble implies ridiculous behavior for larger-stakes gambles.

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$$\begin{aligned}
(12, .5; -10, .5) &\sim (0, 1)\\
(120, .5; -100, .5) &\sim (0, 1)\\
(1200, .5; -1000, .5) &\sim (0, 1)
\end{aligned}$$

Application #3: The Equity-Premium Puzzle

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- The equity premium is quite large: For instance, since 1926, the real return on stocks has been about 7%, and the real return on T-Bills has been about 1%.
- The puzzle: The equity premium is “too large” — they estimate that investors would need to have absurd levels of risk aversion to explain the historical equity premium.

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- Hence, any risk in your financial portfolio gets translated into risk in future consumption.
- And therefore any risk aversion with regard to future consumption gets translated into risk aversion with regard to your financial portfolio.

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  1. Given the historical equity premium, under EU (and CRRA utility) people's observed willingness to hold a mix of stocks and bonds can be explained only by a $\rho > 30$, which is clearly absurd (i.e., it would imply absurd behavior in other domains).
  2. Given EU and reasonable levels of risk aversion ($\rho = 1$ or perhaps even $\rho = 5$), under the historical equity premium, we should observe people investing exclusively in stocks.

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- Two components: loss aversion and a specific mental-accounting assumption.
- Basic hypothesis: From time to time, a person evaluates her portfolio and experiences joy/pain from watching it grow/shrink.
- This is akin to changing the “Samuelson Colleague” example so the person feels the gain/loss from each individual coin flip instead of integrating the 100 coin flips together

Benartzi & Thaler's Model: Objects for Evaluation

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$$t, t + \triangle, t + 2\triangle, ...$$

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- At date τ, person chooses between lotteries over $x_{\tau+\Delta}$.
- Key idea: The person's portfolio allocation chosen at date τ generates a lottery over $x_{\tau+\Delta}$ — that is, a lottery over how her portfolio will change in value between now (τ) and the next evaluation period ($\tau + \Delta$).

  $\Rightarrow$ Picking portfolio to optimize how you'll feel when you check it next, not how you'll feel at retirement

Benartzi & Thaler's Model: Objects for Evaluation

- Much simplified example: Suppose there are two assets, stocks and bonds, and that between τ and $\tau + \Delta$ the returns are:
  - For bonds: $(+1\%, 1)$
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- Suppose further that the person must choose a proportion α of her wealth to invest in stocks, with the remainder invested in bonds. As a function of α, the resulting lottery over $x_{\tau+\Delta}$ is

$$\left(\alpha w(.10) + (1-\alpha) w(.01) \ , \ \frac{1}{2} \ ; \ \alpha w(-.05) + (1-\alpha) w(.01) \ , \ \frac{1}{2} \right)$$

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- Again, at date τ, person chooses between lotteries over $x_{\tau+\Delta}$.
  - Note: Major deviation from the standard approach!

Benartzi & Thaler's Model: Evaluating Lotteries

- At date t, person chooses her portfolio to maximize her "prospective utility"

$$\sum_{x_{t+\Delta}} \pi(x_{t+\triangle}) v(x_{t+\triangle}).$$

- Value function is

$$v(x) = \begin{cases} x^{\alpha} & \text{if } x \geq 0 \\ -\lambda(-x)^{\beta} & \text{if } x \leq 0 \end{cases}$$

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- $\pi(x_{t+\triangle})$ reflects probability weighting

Benartzi & Thaler's Analysis: Simulations

General approach:

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    then for that asset they'd set $\Pr(1\%) = 0.4$, $\Pr(0\%) = 0.3$, etc.

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- Then, consider n-month evaluation periods, for $n = 1, 2, 3, \ldots$, where the distribution of returns for an n-month evaluation period is constructed from n IID draws from the distribution of monthly returns.
  - E.g., if the monthly return distribution is $(20\%, 1/2; 0\%, 1/2)$, then the 2-month return distribution is $(44\%, 1/4; 20\%, 1/2; 0\%, 1/4)$.

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First simulation:

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- What evaluation period n would make investors indifferent between holding all stocks vs. holding all bonds?
- Conclusion: The historical data are consistent with their model and people evaluating their portfolios about once a year.

Benartzi & Thaler’s Analysis: Simulations

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- Assume yearly evaluations — what is the optimal mix of stocks and bonds?

Benartzi & Thaler's Analysis: Simulations

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- Assume yearly evaluations — what is the optimal mix of stocks and bonds?
- Conclusion: The historical data and yearly evaluations are consistent with people investing roughly equally in stocks and bonds (as we observe in the world).

Next Class

Today: Applications of Prospect Theory — the Samuelson bet, risk aversion, and the equity premium puzzle

Next class: More applications of Prospect Theory

Remember:

- Problem Set 2 is due Thursday, September 24 — keep chipping away at it
- Exam 1 is Tuesday, September 29 (in class)