Behavioral Economics

Topic 1: Choice under Uncertainty

Lecture 3: Expected Utility, concluded; the Evidence Against It begins

Professor Bushong

MICHIGAN STATE UNIVERSITY

Taking Stock

Last class:

- Introduction to choice under uncertainty
- Expected value, expected utility, and the calculus toolkit

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Logistics:

- Problem Set 1 is posted – due Tuesday, September 15
    - Tip: start early – don't wait until the last minute
    - You should be ready for most problems after today's class

Expected Utility Theory, Continued

What's Missing from EV? $\rightarrow$ "Risk Aversion"

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Note:

- Risk neutral person always picks the lottery with highest EV
- Risk averse person faces a trade-off: they like higher EV but dislike variance

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Reminders:

- We'll implicitly assume "more is better"
- An EU maximizer never chooses a "dominated" lottery
- Integration with existing wealth: Proper way to write the definition above is to acccount for existing wealth — if person starts with wealth w, then the expected utility from $\mathbf{x}$ is

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- For $\rho = 0$, equivalent to $u(x) = x$, or risk neutrality.
- $\rho > 0$ implies risk aversion
- Example: suppose wealth $w = 100$ and consider $x = (40, \frac{1}{2}; 0, \frac{1}{2})$ vs $y = (120, \frac{1}{3}; -30, \frac{2}{3})$
  - Which will person pick if risk neutral (i.e., an expected value maximizer)?
  - Which will person pick if EU maximizer with CRRA utility and $\rho = \frac{1}{2}$? (Note: answering by computing EU of each is fine, but can actually figure out answer without doing so in this special case!)

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Brief Calculus Refresher

Application: Demand for Financial Assets

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- If do well, final wealth $= (1000 - y)(1.00) + y(1.50) = 1000 + (.50)y$.
- If do poorly, final wealth $= (1000 - y)(1.00) + y(0.60) = 1000 - (.40)y$.

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$$EU(\mathbf{x}(y)) = \frac{1}{2}\,u(1000 + (.50)y)\ +\ \frac{1}{2}\,u(1000 - (.40)y).$$

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- **How to solve:** (i) take derivative of expression for EU with respect to y; (ii) set the derivative equal to 0 and then solve equation for y

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- **How to solve:** (i) take derivative of expression for EU with respect to y; (ii) set the derivative equal to 0 and then solve equation for y
- **Answer:** Optimal choice is $y^* = 250$

Application: Demand for Insurance

Recall insurance example from last class:

You have wealth \$1000, and there's a 10% chance that you'll suffer a loss of \$250.

- Hence, you face the lottery (1000, .9 ; 750, .1).

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What is your willingness to pay for full insurance?

- Answer: It's the p^* such that

$$(\ 1000 - p^*, 1\ ) \sim (\ 1000, .9\ ;\ 750, .1\ )$$

$$\Rightarrow \quad u(1000 - p^*) = (.9)\ u(1000) + (.1)\ u(750).$$

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- Step 1: Derive final lottery as a function of α.
- Step 2: Calculate expected utility as a function of α.
- Step 3: Use calculus to solve for the optimal α.

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As a function of the α you choose, final lottery is:

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- Here, actuarially fair insurance means $p = (.10)(250) = \$25$.

Side Note:

- In general„ actuarially fair price is the highest amount a risk-neutral person (i.e., EV maximizer) is willing to pay for any insurance
- A risk averse person would be willing to pay more to avoid the risk

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Now we need to do some calculus!

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. . . but first let's use our brains a bit more.

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And likewise:

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Notice anything?

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- Point: The level of utility doesn't matter, only the comparisons.

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- If they have perfect positive correlation, the reduced lottery is

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Four More Features/Issues of Expected Utility

(3) *EU* operates on *reduced lotteries*.

Example: If you face both lottery $\mathbf{x} = (100, 1/2; -90, 1/2)$ and $\mathbf{y} = (95, 1/2; -85, 1/2)$, you must first combine these into a reduced lottery before applying *EU*.

- If they are independent, the reduced lottery is

$$(\, 195, 1/4 \; ; \; 15, 1/4 \; ; \; 5, 1/4 \; ; \; -175, 1/4 \,).$$

- If they have perfect positive correlation, the reduced lottery is

$$(\, 195, 1/2 \; ; \; -175, 1/2 \,).$$

- If they have perfect negative correlation, the reduced lottery is

$$(\, 15, 1/2 \; ; \; 5, 1/2 \,).$$

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Question: How to formalize "more risk averse"?

Definition: The *coefficient of risk aversion* at wealth w is

$$r(w) = \frac{-u''(w)}{u'(w)}.$$

Suppose w_A is person A's wealth and w_B is person B's wealth. If $r(w_A) > r(w_B)$ then person A is less likely than person B to accept a risky gamble.

Four More Features/Issues of Expected Utility

Note: A person's risk aversion depends on both her utility function and her wealth.

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How is one's willingness to accept a gamble likely to depend on my wealth?

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How is one's willingness to accept a gamble likely to depend on my wealth?

- Economists often <u>assume</u> risk aversion decreases with wealth:

$$\uparrow w \Longrightarrow \downarrow r(w).$$

An Important Functional Form

Definition: The *CRRA utility function* is

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$$\text{gamble } (\$11, 1/2; -\$10, 1/2) \text{ from wealth } \$1000$$

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Expected Utility and Revealed Preferences

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- (2) Preferences are continuous.
- (3) Independence axiom: If $\mathbf{x} \succsim \mathbf{y}$, then for any lottery $\mathbf{z}$, $(1-\alpha)\mathbf{x} + \alpha\mathbf{z} \succsim (1-\alpha)\mathbf{y} + \alpha\mathbf{z}$ for any $0 < \alpha < 1$.

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<u>Theorem</u>: A person's behavior can be described by expected utility theory if and only if her choices over lotteries are complete, transitive, continuous, and satisfy the independence axiom.

Expected Utility – A Final Thought

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However, *EU* seems to fail as a positive (descriptive) theory (i.e., a way that people *actually* behave). That's where we'll go next....

Evidence That Contradicts the Standard Model

Allais Paradox (Allais 1953)

Question 1:

Option (A)		Option (B)	
\$1 million	with prob. 1	\$1 million	with prob. 0.89
		\$5 million	with prob. 0.10
		\$0	with prob. 0.01

Question 2:

Option (C)		Option (D)	
\$1 million	with prob. 0.11	\$5 million	with prob. 0.10
\$0	with prob. 0.89	\$0	with prob. 0.90

- The Paradox: The combination of choosing A over B and choosing D over C violates expected utility.

Allais Paradox (Revealed)

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Option (A)			Option (B)		
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Question 2:

Option (C)			Option (D)		
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$1 million	with prob.	0.11	$5 million	with prob.	0.10
			$0	with prob.	0.01

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Option A: You win $100 if the ball is red. Option B: You win $100 if the ball is black.

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Question 1:

Option A: You win \$100 if the ball is red. Option B: You win \$100 if the ball is black.

Question 2:

Option C: You win \$100 if the ball is either red or yellow.

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Option C: You win $100 if the ball is either red or yellow.

Option D: You win $100 if the ball is either black or yellow.

- The Paradox: The combination of choosing A over B and choosing D over C violates expected utility — in particular, violates that people form stable subjective beliefs.

Evidence from Kahneman & Tversky (1979)

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"Subproportionality"

Problem 3:	Option (A)	vs.	Option (B)
$[N = 95]$	$(4000, .8)$		$(3000, 1)$

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Problem 4:	Option (C)	vs.	Option (D)
$[N = 95]$	$(4000, .2)$		$(3000, .25)$

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From these and similar examples, Kahneman & Tversky conclude that preferences exhibit “subproportionality”:

- When comparing a larger/less-likely reward to a smaller/more-likely reward, if we scale down the probabilities proportionally, the person becomes more and more likely to choose the larger/less-likely reward.

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- Formally: If $(y, pq) \sim (x, p)$ then $(y, pqr) \succ (x, pr)$.
  [where $y > x > 0$ and $p, q, r \in (0, 1)$].

“Reflection Effect”

Problem 7:	Option (A)	vs.	Option (B)
$[N = 66]$	(6000, .45)		(3000, .90)

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“Reflection Effect”

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Problem 7′:	Option (C)	vs.	Option (D)
$[N = 66]$	$(-6000, .45)$		$(-3000, .90)$

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“Isolation Effect”

Problem 10: Consider the following two-stage game. In the first stage, there is a probability of .75 to end the game without winning anything, and a probability of .25 to move into the second stage. If you reach the second stage you have a choice between

$$(4000, .80) \qquad \text{and} \qquad (3000, 1).$$

Your choice must be made before the game starts, i.e., before the outcome of the first stage is known.

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Note: we can collapse this to

Problem 10: $[N = 141]$	$(4000, .2)$	$(3000, .25)$

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Problem 3:	$(4000, .8)$	$\prec$	$(3000, 1)$
$[N = 95]$	[20%]		[80%]*
Problem 4:	$(4000, .2)$	$\succ$	$(3000, .25)$
$[N = 95]$	[65%]*		[35%]

"Isolation Effect"

Problem 11: You get 1000 for sure. In addition, choose between $[N = 70]$

$$(1000, .5) \qquad \text{vs.} \qquad (500, 1)$$

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$$(1000, .5) \qquad \text{vs.} \qquad (500, 1)$$

Problem 12: You get 2000 for sure. In addition, choose between $[N = 68]$

$$(-1000, .5) \qquad \text{vs.} \qquad (-500, 1)$$

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Brief aside: There is now a large literature on “framing effects” — two ways of presenting the <u>**exact same problem**</u> elicit different choices.

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Brief aside: There is now a large literature on “framing effects” — two ways of presenting the **exact same problem** elicit different choices.

The isolation effect is a natural interpretation of some framing effects — because for some ways of framing a problem, certain information can seem extraneous.

Next Class

Today: Finished EU and started looking at evidence contradicting that model

Next class: More evidence that contradicts the standard model – and a first look at where the fix will come from

Remember:

- Problem Set 1 is due Tuesday, September 15
- Problem Set 2 posts Thursday – it is long *by design*, so start early