

Behavioral Economics

Lecture 2: Making Choices over Time Part (c): Evidence that Contradicts Exponential Discounting

Professor Bushong



General Version of Discounted Utility



Recall the general version of the discounted-utility model:

$$U^t = \sum_{x=0}^{T-t} D(x) u_{t+x}.$$

- U^t is intertemporal utility from perspective of period t .
- u_τ is instantaneous utility in period τ (“well-being” in period t).
- x is the delay before receiving some utility.
- $D(x)$ is a discount function that specifies the amount of discounting associated with delay x .

Again, in principle, we could have any discount function.

Exponential discounting **assumes** $D(x) = \delta^x$.

How to Measure Discount Functions?



Typical procedure elicits indifference points of the form:

$$(A \text{ at date } x) \sim (B \text{ at date } x + y)$$

To interpret, typically make four assumptions:

■ “The Usual Assumptions”

- (1) People obey discounted utility model.
- (2) People treat these amounts as “bursts” of consumption.
- (3) “Utility” is linear in the amount.
- (4) Normalize $D(0) = 1$.

How to Measure Discount Functions?



An implication of the usual assumptions:

$$(A \text{ now}) \sim (B \text{ at date } x)$$

$$\iff D(0)u(A) = D(x)u(B)$$

$$\iff A = D(x)B$$

$$\iff D(x) = \frac{A}{B}$$

How to Measure Discount Functions?



More generally, the usual assumptions imply:

$$(A \text{ at date } x) \sim (B \text{ at date } x + y)$$

$$\iff D(x)u(A) = D(x + y)u(B)$$

$$\iff D(x)A = D(x + y)B$$

$$\iff \frac{D(x + y)}{D(x)} = \frac{A}{B}$$

How to Measure Discount Functions?



An alternative procedure elicits WTP now for something to be received later — e.g., if WTP up to A now to obtain B at date x , this implies:

$$(-A \text{ now \& } +B \text{ at date } x) \sim (\text{no changes})$$

Applying the usual assumptions here yields:

$$D(0)u(-A) + D(x)u(B) = 0$$

$$\Longleftrightarrow D(x)B = A$$

$$\Longleftrightarrow D(x) = \frac{A}{B}$$

Some Evidence from Thaler (1981)



Using a (hypothetical) matching technique, Thaler found that people were indifferent between:

- a. (\$15 now) $\sim$ (\$30 in 3 months)
- b. (\$15 now) $\sim$ (\$60 in 1 year)
- c. (\$15 now) $\sim$ (\$100 in 3 years)

Implications under the usual assumptions:

- a. $D(3 \text{ months}) = \frac{15}{30} = 0.50$.
- b. $D(1 \text{ year}) = \frac{15}{60} = 0.25$.
- c. $D(3 \text{ years}) = \frac{15}{100} = 0.15$.

Some Evidence from Thaler (1981)



We can convert each $D(x)$ into an implicit yearly discount rate:

Definition: The *average yearly discount rate applied to delay x* (where x is specified in years) is the ρ such that

$$e^{-\rho x} = D(x) \quad \text{or} \quad \rho = \frac{1}{x}(-\ln D(x)).$$

Applying this definition:

- a. $D(3 \text{ months}) = 0.50 \implies 277\%$ yearly discounting.
- b. $D(1 \text{ year}) = 0.25 \implies 139\%$ yearly discounting.
- c. $D(3 \text{ years}) = 0.15 \implies 63\%$ yearly discounting.

More Evidence from Thaler (1981)



Thaler (1981) also found that people were indifferent between:

e. (\$250 now) $\sim$ (\$300 in 3 months)

f. (\$250 now) $\sim$ (\$350 in 1 year)

g. (\$250 now) $\sim$ (\$500 in 3 years)

Implications under the usual assumptions:

■ e. $D(3 \text{ months}) = \frac{250}{300} = 0.83 \implies 73\%$ yearly discounting.

■ f. $D(1 \text{ year}) = \frac{250}{350} = 0.71 \implies 34\%$ yearly discounting.

■ g. $D(3 \text{ years}) = \frac{250}{500} = 0.50 \implies 23\%$ yearly discounting.

Conclusions from Thaler (1981)



Conclusion #1: The amount matters — there is more discounting for smaller amounts (“magnitude effect”).

But the key conclusion is:

Conclusion #2: For either amount, discount rates are higher in the short run than in the long run (sometimes referred to as “declining discount rates”).

- This finding is inconsistent with exponential discounting!

Some “Hypothetical Evidence” on “Preference Reversal”



Consider the following hypothetical choice scenarios:

Choice 1:

[10 M&M's now] vs. [15 M&M's tomorrow]

Choice 2:

[10 M&M's in 7 days] vs. [15 M&M's in 8 days]

A plausible pattern:

(10 M&M's now) $\succ$ (15 M&M's tomorrow)

(10 M&M's in 7 days) $\prec$ (15 M&M's in 8 days)

Some “Hypothetical Evidence” on “Preference Reversal”



Implication: Under the usual assumptions, this pattern implies

$$D(0)_{10} > D(1)_{15} \iff \frac{D(0)}{D(1)} > 1.5$$

$$D(7)_{10} < D(8)_{15} \iff \frac{D(7)}{D(8)} < 1.5$$

Hence: $\frac{D(0)}{D(1)} > \frac{D(7)}{D(8)}$

Conclusion: If you exhibit this pattern, then you are more impatient toward the now-vs.-near-future tradeoff than you are toward the near-future-vs.-further-future tradeoff.

Some Real Evidence on “Preference Reversals”



In fact, Kirby & Herrnstein (1995) show that, for 23 of their 24 subjects, they can make the subject exhibit this type of “preference reversal”.

For instance, for each of these 23 subjects, they find an $x > 0$ and a $y > 0$ such that the subject's preferences are

$$(\$45 \text{ now}) \succ (\$52 \text{ in } y \text{ days})$$

$$(\$45 \text{ in } x \text{ days}) \prec (\$52 \text{ in } x + y \text{ days})$$

Much as above, under the usual assumptions, this implies

$$\frac{D(0)}{D(y)} > \frac{D(x)}{D(x+y)}$$

Again, more impatient toward the now-vs.-near-future tradeoff than toward the near-future-vs.-further-future tradeoff.

Evidence of “Hyperbolic Discounting”



Another approach to the same type of data is to directly compare two functional forms:

- Exponential Discounting: $D(x) = e^{-kx}$
- Hyperbolic Discounting: $D(x) = \frac{1}{1+kx}$

In these comparisons, the answer is that hyperbolic discounting is virtually always a better fit (occasionally, they're equally good).

Evidence of “Hyperbolic Discounting”



For instance, Kirby (1997) elicited WTP's for \$20 to be received in x days, where each subject answered for every odd x between 1 and 29.

He then tested for each subject whether their discount function was better fit by the exponential functional form or the hyperbolic functional form.

Results: Hyperbolic was a better fit for 59 of 67 subjects, exponential was a better fit for 6 subjects, and for 2 subjects the functions were equally good.

Conclusions from the Evidence



In the experimental data, there seems to be a key feature that virtually always holds:

- Discount rates are higher in the short run than in the long run (sometimes referred to as “declining discount rates”).

In terms of our notation, the evidence seems to suggest:

$$\frac{D(0)}{D(1)} > \frac{D(1)}{D(2)} > \frac{D(2)}{D(3)} > \dots$$

Note: This suggests a time inconsistency of an impulsive nature:

- When thinking about some future period, you'd like to behave relatively patiently; but when the time comes to actually choose your behavior, you want to behave relatively impatiently.