Choice over Time: Introduction

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Topic 2:

Intertemporal Choice or Making Choices Over Time

Many interesting questions in economics involve choice over time:

- How do people allocate their wealth between current consumption and future consumption?
- How do people decide when to work on tasks?
- For goods that yield short-term consumption utility but generate negative consequences in the long-term---e.g., alcohol, cigarettes, potato chips---how do people trade off the short-term benefits vs. the long-term costs?

Overview of Topic 2

The standard model ("exponential discounting") assumes:

1. People treat time in a relatively even-handed manner.
2. People carry out their plans.
3. People know what they'll like in the future.

Warm-Up: Interest Rates, Compounding, PDV

Let's understand some of the precursors to the standard model and (quickly) do some example problems. (This is a great time to ask questions.)

The Standard Model (Today + Thursday)

Example A: Suppose you put $1000 into a bank account that pays 10% interest per year.

- After 1 year, you'll have $1000 * (1.10) = $1100 .
- After 2 years, you'll have $1100 * (1.10)= $1210 .
- After 3 years, you'll have $1210 * (1.10)= $1331 .

More generally:

If you put P into a bank account that pays interest rate r per year, its future value in T years will be $P * (1+r)^T$.

Definitions (easy; hopefully not new)

Compound interest is interest paid on past interest earned.

Compounding is earning interest on past interest earned.

The **frequency of compounding** is the frequency at which interest is credited to your account (after which it's starts earning compound interest).

Our example above implicitly assumed yearly compounding. Of course, we could have more frequent compounding....

Example B: Suppose you put $1000 into a bank account that pays a 10% annual interest rate that is compounded every six months.

Because a 10% annual interest rate implies a 5% semi-annual interest rate:

- After 6 months, you'll have $\mathbf{1000 * (1.05) = 1050}$.
- After 1 year, you'll have $\mathbf{1050 * (1.05) = 1102.50}$.

Example C: Suppose you put $1000 into a bank account that pays a 10% annual interest rate that is compounded every month.

Because a 10% annual interest rate implies a $\mathbf{0.8\bar{3}}$ % monthly interest rate:

- After 1 year, you'll have $\mathbf{(1000) * (1.008\bar{3})^{12} = 1104.71}$.

More generally, if you put P into a bank account that pays an annual interest rate of r that is compounded n times per year:

- Its future value after 1 year will be $(P) * (1 + r/n)^n$.
- Its future value after T years will be $(P) * \left[(1 + r/n)^n\right]^T$.
- Note: For continuous compounding, $\lim_{n\to\infty}(1 + r/n)^n = e^r$ and $\lim_{n\to\infty}\left[(1 + r/n)^n\right]^T = e^{rT}$.

Suppose there is some set of periods $0, 1, 2, \ldots, T$ (perhaps $T = \infty$).

- Note: The length of a period might be one year, one month, one day, or whatever is most appropriate for the particular application.

Suppose there is a per-period interest rate r, and interest is compounded every period.

If P_t is the principal in your bank account in period t, then:

- $P_1 = (1 + r) * P_0$
- $P_2 = (1 + r)^2 * P_0$
- $P_t = (1 + r)^t * P_0$
- $P_6 = (1 + r) * P_5$
- $P_6 = (1 + r)^4 * P_2$
- $P_{t+x} = (1 + r)^x * P_t$

Suppose that you will be paid \$1100 one year from today. If the market interest rate is 10% (and yearly compounding), how much is this future payment be worth to you now?

We can answer this question by asking how much you could borrow now such that you would have to pay back exactly \$1100 in one year.

- Answer: \$1000 --- because $(1.10) * (1000) = 1100$.

Definition: Given per-period interest rate r, the *present discounted value* (or sometimes just *present value* or *PDV*) of P to be paid T periods in the future is

$$\frac{P}{(1+r)^T}$$

Some PDV 's for $P = 1000$ and yearly compounding:

r	1 Year	2 Years	3 Years	10 Years	20 Years
3%	$971	$943	$915	$744	$554
4%	$962	$925	$889	$676	$456
5%	$952	$907	$864	$614	$377
6%	$943	$890	$840	$558	$312
7%	$935	$873	$816	$508	$258

Suppose that you will be paid \$1100 one year from today, another \$1100 two years from today, and yet another \$1100 three years from today.

If the market interest rate is 10% (and yearly compounding), how much is this stream of payoffs worth to you now?

Answer: Add up the individual PDV bit-by-bit:

$$PDV = \frac{\$1100}{(1.10)} + \frac{\$1100}{(1.10)^2} + \frac{\$1100}{(1.10)^3} = \$2735.54.$$

More generally: Given per-period interest rate r, a stream of future revenues $(R_1, R_2, \ldots, R_N)$ (where revenue R_n is received in period n) has a present discounted value of:

$$PDV = \frac{R_1}{(1+r)} + \frac{R_2}{(1+r)^2} + \ldots + \frac{R_N}{(1+r)^N}.$$

Definition: The *period-* t *interest rate* r_t is the interest rate between period t and period $t+1$. In other words, if in period t your principal is P_t, then in period $t+1$ it becomes $P_{t+1} = (1+r_t)P_t$.

Hence, if P_t is the principal in your bank account in period t, and if your bank account pays per-period interest rates $(r_t, r_{t+1}, \dots)$, then:

- $P_{t+1} = (1+r_t)P_t$.
- $P_{t+2} = (1+r_{t+1})P_{t+1} = (1+r_{t+1})(1+r_t)P_t$.
- $P_{t+3} = (1+r_{t+2})P_{t+2} = (1+r_{t+2})(1+r_{t+1})(1+r_t)P_t$.
- And so on....

Given per-period interest rates $(r_t, r_{t+1}, \dots)$, a stream of future revenues $(R_{t+1}, R_{t+2}, R_{t+3})$ has a present discounted value of

$$PDV = \frac{R_{t+1}}{(1+r_t)} + \frac{R_{t+2}}{(1+r_t)(1+r_{t+1})} + \frac{R_{t+3}}{(1+r_t)(1+r_{t+1})(1+r_{t+2})}.$$

End of Warm-Up

As always, you will *not* need to memorize any of these equations. But we want to remind ourselves how to think about choices across time.

- How do you assess today versus tomorrow?
- What is the "correct" weight to put on money today versus money tomorrow?
- What determines this tradeoff?

We'll explore these questions (and many more) coming up.