Choice under Uncertainty (Lecture 1d)

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Our Outline:

#3: Take Refuge in Simplicity

- We often get lost in mathematics or in what seem like complex ideas. I provide mathematical definitions for *precision*, but this can become overwhelming.

- **Break the ideas down to their components** then work through the idea bit-by-bit.

E.g., the prospect theory mathematics can be confusing, but the "diminishing sensitivity" property of the value function implies that an agent is:

1. Risk averse over only gains.
2. Risk loving over only losses.

(And you remember these definitions... right?)

A person evaluates a prospect $(x_1, p_1; \ldots; x_n, p_n)$ according to:

$$V(x_1, p_1; \ldots; x_n, p_n) \quad = \sum_{i=1}^{N} \pi(p_i)\, v(x_i)\,.$$

Notable features:

- Value comes from **changes** in wealth, not absolute wealth
- $\pi(p) \neq p$ implies people "mess up" probabilities
- Otherwise similar to expected utility. **Not** radical!

A commonly assumed form of $v(\cdot)$ is

$$v(x) = \{x^{\alpha} \quad \text{if} \quad x \geq 0$$

$$\lambda(x)^{\beta} \quad \text{if} \quad x \leq 0$$

and we'll often just drop the exponents.

For many years, expected utility has been used by economists to capture risk preferences. Indeed, it is still used in almost all applications.

But economists are starting to recognize that some behaviors are hard to interpret in terms of expected utility; and for many such behaviors, prospect theory provides a natural interpretation.

To illustrate, we'll consider nine examples.

- I suspect much more work in this area in coming years.
- Only one example addresses probability weighting -- but I suspect it is running around in tons of seemingly-strange behaviors.

Example courtesy of Samuelson (1963)

Consider the following bet:

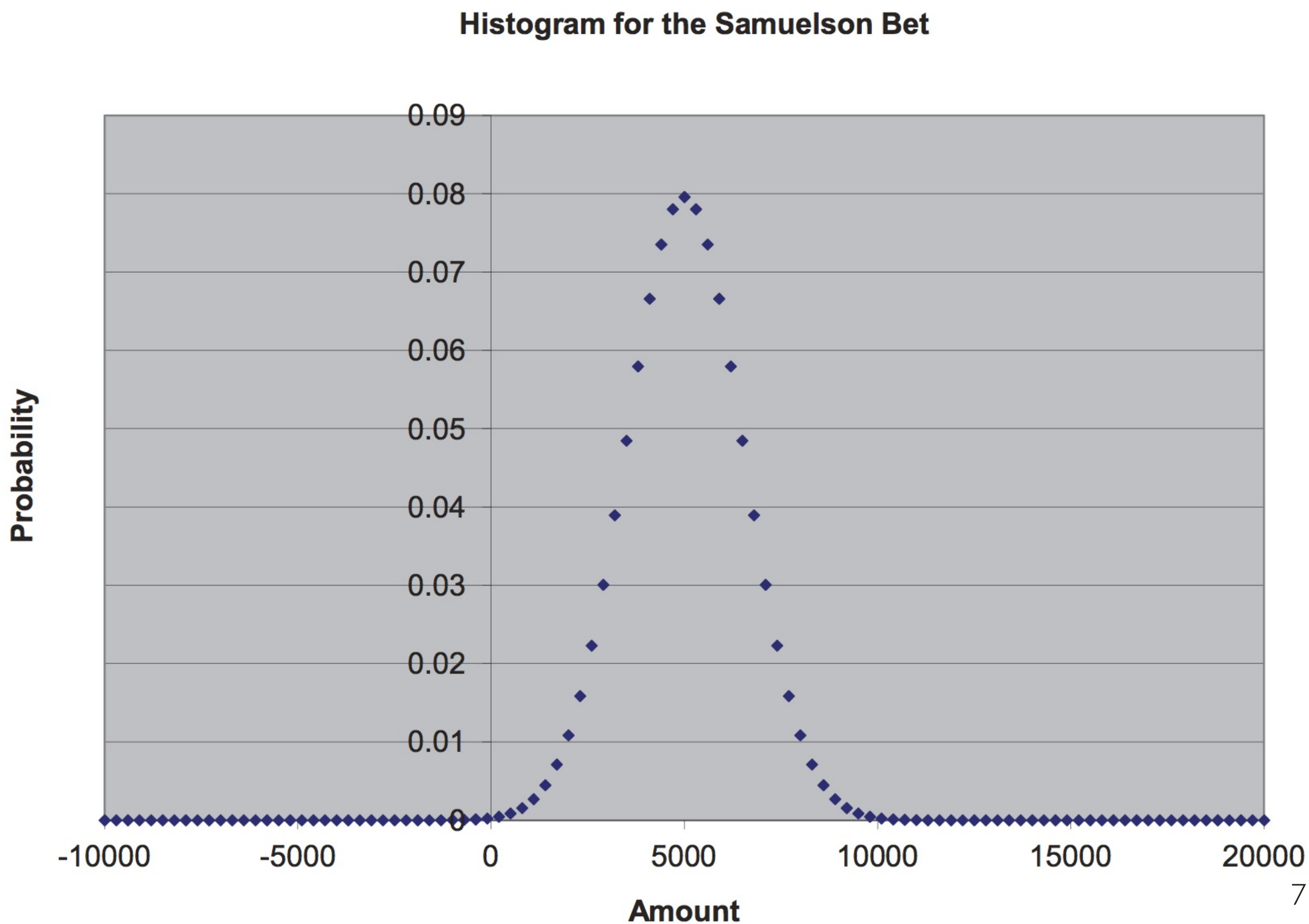
$$\textbf{win \$200 with prob } \mathbf{1/2}$$

$$\textbf{lose \$100 with prob } \mathbf{1/2}$$

Samuelson's colleague turned down this bet, but announced that he would accept 100 plays of the same bet.

Samuelson proved that his colleague was "irrational" --- by proving that it is inconsistent with expected-utility theory to turn down the single bet but accept 100 such bets.

But **was** his colleague "irrational"?


Histogram for the Samuelson Bet
Probability
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
-10000
-5000
0
5000
10000
15000
20000
Amount

Class discussion: Suppose that a person turns down the bet **at some wealth levels.** Does EU imply that the person must turn down 100 such bets?

Continued: Suppose that a person turns down the bet **at all wealth levels**. Does EU imply that the person must turn down 100 such bets?

What is the basic intuition?

Intuition: Consider an individual who has said that he is unwilling to take one bet but is willing to play 100 such bets. Suppose this person has played 99 bets.

- If asked whether he would like to stop at this point he will say yes. By assumption, he dislikes one bet at any relevant wealth level.
- However, this means that if asked after 98 bets whether he would like to play number 99 he must also decline.
- He should realize (by backward induction) that he would reject bet 100, implying that bet 99 is a single play.

The same reasoning applies to the first bet.

Thinking about economics grad school? Prove this claim formally.

Consider an alternative "model":

- Suppose that a person evaluates bets according to the value function

$$v(x) = x \quad \text{if} \quad x \geq 0$$

$$v(x) = 2.5x \quad \text{if} \quad x \leq 0$$

Consider the single bet $y = [200, .5; -100, .5]$.

Consider taking two such bets. This means you face aggregate gamble $z = [400, .25; 100, .5; -200, .25]$.

Point: Unlike EU, loss aversion can lead a person to reject one play of the bet but to accept multiple plays of the bet.

Note: The Previous Explanation Was Underspecified

Mental accounting: the process a person uses to interpret a choice situation.

- **Any** application of prospect theory requires a mental-accounting assumption.
- Typically, this requires an assumption about how people decide what are the objects for evaluation.
- E.g., Kahneman & Tversky interpret the isolation effect as people ignoring seemingly extraneous parts of the problem.
- E.g., to explain the behavior of Samuelson's colleague, we assumed that the person collapses the aggregate bet into a single lottery and decides whether to accept that lottery.
- Sometimes, we must make an assumption about when and how people code outcomes as gains and losses. (We'll do this in later applications)

People tend to dislike risky prospects even when they involve an expected gain.

Rabin & Thaler's point, which should feel **very** repetitive by now:

- Calibrationwise, this explanation doesn't work, because according to EU, "anything but virtual risk neutrality over modest stakes implies manifestly unrealistic risk aversion over large stakes."
- Now we'll show that *loss aversion* is a useful alternative.

Suppose you have wealth $20,000, and you turn down a 50-50 bet to win $110 vs. lose $100.

We showed that rejecting the bet implies that $\rho > 18.17026$.

Question: What about with loss aversion?

Two plausible features of preferences consistent with loss aversion:

1. How you feel about absolute gambles is largely insensitive to your wealth --- e.g., you might reject $(101, .5; -100, .5)$ for all w.
2. At the same time, scaling outcomes proportionally need not change your preferences much --- e.g., you might have

$$(12, .5; -10, .5) \sim (0, 1)$$

$$(120, .5; -100, .5) \sim (0, 1)$$

$$(1200, .5; -1000, .5) \sim (0, 1)$$

Equity-Premium Puzzle (Mehra & Prescott, 1985)

Equity premium: The difference between the returns on stocks and the returns on fixed-income securities.

- The (historical) equity premium is quite large. For instance, since 1926, the real return on stocks has been about 7%, and the real return on T-Bills has been about 1%.

The puzzle: The equity premium is "too large" --- Mehra and Prescott estimate that investors would need to have absurd levels of risk aversion to explain the historical equity premium.

Standard Economics/Finance View of Financial Decisions

You have wealth w, and you use this wealth for your lifetime consumption profile $(c_1, c_2, \ldots, c_T)$.

Your lifetime consumption profile yields lifetime utility

$$u(c_1) + \delta u(c_2) + \delta^2 u(c_3) + \ldots + \delta^{T-1} u(c_T)$$

Wealth that's targeted for future consumption is invested in financial assets (stocks and bonds).

- Hence, any risk in your financial portfolio gets translated into risk in future consumption.
- And therefore any risk aversion that have with regard to future consumption gets translated into risk aversion with regard to your financial portfolio.

Assume a CRRA utility function (over consumption):

$$u(c) = \frac{(c)^{1-\rho}}{1-\rho}.$$

- *Note:* The larger is ρ, the **less** risk one takes on in one's financial portfolio (fewer stocks, more bonds).
- Mehra & Prescott show that to explain the observed equity premium, we need to assume that people have $\rho > 30$.
- But empirical estimates and theoretical arguments suggest $\rho \approx 1$ (log utility) and definitely not more than 5. (Remember the exercise we did?)

Two Interpretations for Mehra and Prescott Result

- Given the historical equity premium, under EU (and CRRA utility) people's observed willingness to hold a mix of stocks and bonds can be explained only by a $\rho > 30$, which is clearly absurd (i.e., it would imply absurd behavior in other domains).
- Given EU and reasonable levels of risk aversion ($\rho = 1$ or perhaps even $\rho = 5$), under the historical equity premium, we should observe people investing exclusively in stocks.

Benartzi & Thaler's explanation: "Myopic Loss Aversion"

- Two components: loss aversion and a specific mental-accounting assumption.

Basic foundation: From time to time, a person evaluates her portfolio and experiences joy/pain from watching it grow/shrink.

Objects for Evaluation

Suppose a person evaluates her portfolio at dates

$$t, t + \triangle, t + 2\triangle, \ldots$$

Let Y_τ be the value of her portfolio at date τ.

Let $x_{\tau+\Delta} \equiv Y_{\tau+\Delta} - Y_\tau$.

At date τ, person chooses between lotteries over $x_{\tau+\Delta}$.

Key idea: The person's portfolio allocation chosen at date τ generates a lottery over $x_{\tau+\Delta}$ --- that is, a lottery over how her portfolio will change in value between now (τ) and the next evaluation period ($\tau + \Delta$).

A (Much) Simplified Example:

Suppose there are two assets, stocks and bonds, and that between τ and $\tau + \Delta$ the returns are:

- For bonds: $(+1\%, 1)$
- For stocks: $(+10\%, \frac{1}{2}; -5\%, \frac{1}{2})$

Suppose further that the person must choose a proportion α of her wealth to invest in stocks, with the remainder invested in bonds. As a function of α, the resulting lottery over $x_{\tau+\Delta}$ is

$$\textbf{Good Outcome:} \qquad \alpha w(.10) + (1-\alpha)w(.01)\ ,\ \frac{1}{2}$$

$$\textbf{Bad Outcome:} \qquad \alpha w(-.05) + (1-\alpha)w(.01)\ ,\ \frac{1}{2}$$

Again, at date τ, person chooses between lotteries over $x_{\tau+\Delta}$.

> Note: Major deviation from the standard approach!

Evaluating Lotteries

At date t, person chooses her portfolio to maximize her "prospective utility"

$$\sum_{x_{t+\Delta}} \pi(x_{t+\triangle})v(x_{t+\triangle}).$$

Let's use the value function

$$v(x) = x^{\alpha} \qquad \text{if} \quad x \geq 0$$

$$v(x) = -\lambda(-x)^{\beta} \qquad \text{if} \quad x \leq 0$$

The authors assume $\alpha = \beta = .88$ and $\lambda = 2.25$ (Tversky & Kahneman, 1992).

$\pi(x_{t+\triangle})$ reflects probability weighting. The authors use the cumulative form --- including the suggested parameter values --- from Tversky & Kahneman, 1992.

General (Simulation) Approach

1. Draw samples from historical (1926-1990) monthly returns on stocks, 5-yr bonds, and T-Bills.

- E.g., if 10 observations of actual monthly returns on an asset were

$$-2\%, 1\%, 0\%, 1\%, -1\%, 1\%, 2\%, 0\%, 1\%, 0\%$$

... then for that asset they'd set $\Pr(1\%) = 0.4$, $\Pr(0\%) = 0.3$, etc.

1. Then, consider n-month evaluation periods, for $n = 1, 2, 3, \ldots$, where the distribution of returns for an n-month evaluation period is constructed from n IID draws from the distribution of monthly returns.

- E.g., if the monthly return distribution is $(20\%, 1/2; 0\%, 1/2)$, then the 2-month return distribution is $(44\%, 1/4; 20\%, 1/2; 0\%, 1/4)$.

Results

First question: What evaluation period n would make investors *indifferent* between holding all stocks vs. holding all bonds?

Answer: The historical data are consistent with their model applied as if people evaluated their portfolios about once a year.

Second question: Assuming yearly evaluations, what is the **optimal** mix of stocks and bonds?

Answer: The optimal holdings, given the historical data, are to hold roughly equal amounts in stocks and bonds (as we observe in the world).

Snowberg and Wolfers (2010) explore non-linearity in probabilities "in the wild" by investigating horse-race bets.

If Horse A is 2:1 odds this should mean both:

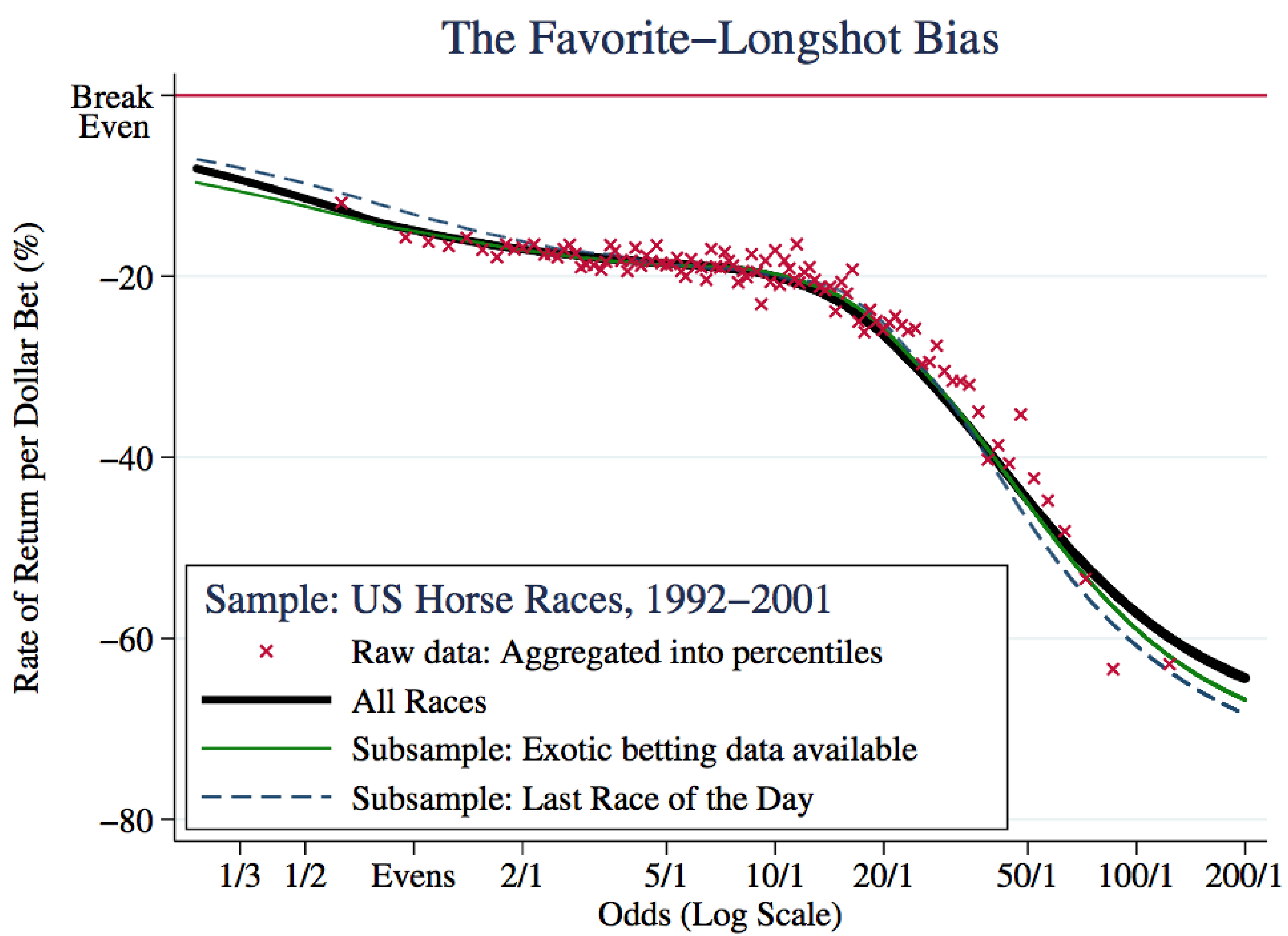
1. That the implied probability of winning is $\frac{1}{3}$. Losing is twice as likely as winning and either the horse wins or loses.
2. If you put \$1 on horse A, you either receive \$3 (\$2 winnings + \$1 stake) or zero.
3. Since $\frac{1}{3} \times 3 + \frac{2}{3} \times 0 = 1$, betting and not betting should yield the same expected return.

(Of course, there is a track profit such that the expected return is a bit negative. But we'll ignore this margin for now.)

Point: If all odds were appropriate, the odds on every horse would correspond to lotteries that all have equal expected value.

Finding: The "Favorite-Longshot Bias"

- Longshots have low expected return, given how rarely they win...
- and bettors value favorites too little given how **often** they win.
- Concretely, betting on a horse with 100/1 odds yields returns of about -61%.
- Betting **randomly** yields average returns of -23%.
- Betting on a horse with 1/3 odds yields returns of only -5.5%



Notes: Sample includes 5,610,580 horse race starts in the United States from 1992–2001. Lines reflect Lowess smoothing (bandwidth=0.4).

"Disposition Effect": When investors sell their stocks, they are more prone to sell their winners than their losers.

- A stock is a "winner" if its current price is above its purchase price, and it is a "loser" if its current price is below its purchase price (as in Shefrin & Statman 1985).

Odean (1998) provides a nice empirical test, and assesses several potential explanations.

Odean (1998) has a dataset of individual traders at a small brokerage house and observes each individual's stock portfolio and all trades made each day.

For every individual-day on which he observes trades, he calculates:

1. "Proportion of Gains Realized":

$$PGR \equiv \frac{\text{\# of winners sold}}{\text{\# of winners in portfolio}}$$

1. "Proportion of Losses Realized":

$$PLR \equiv \frac{\text{\# of losers sold}}{\text{blog\# of losers in portfolio}}$$

"Disposition Effect": $PGR > PLR$.

Big Question: What's the explanation?

Rational Explanation #1: Sell winners to rebalance your portfolio.

- If the disposition effect is driven by rebalancing, then if we restrict attention to trades in which (i) only entire holdings of a stock are sold or (ii) no new purchases are made, we should no longer observe a disposition effect.
- When Odean does this, the effect *does not go away*.

Rational Explanation #2: Sell winners because losers are better.

- Odean finds that the winners people sell *outperform* the losers they keep (over various horizons --- 1/3 year, 1 year, 2 years).

Odean's Explanations

- Loss aversion with a mental-accounting assumption that you experience gain-loss utility for a particular stock when you sell that stock.
- Or an irrational belief in mean reversion.

Based on Genesove and Mayer (2001).

Motivating Example

- Suppose you are offered $400,000 for your house. Do you sell?

The answer clearly depends on many factors --- for instance:

1. How much you like your house.
2. Is your house too large or too small?
3. Do you need to move to a new area?
4. Whether you expect higher offers later.

But should it matter whether you initially paid $350,000 vs. $450,000 when you bought the house?

Genesove and Mayer (2001) analyze data from the Boston condominium market in the 1990's. They compare sellers subject to nominal losses with sellers subject to nominal gains.

They find that people subject to nominal losses:

- set higher asking prices --- roughly, 25-35 percent of the magnitude of the expected loss;
- eventually attain a higher selling price --- roughly, 3-18 percent of the magnitude of the expected loss. That is, would-be "losers" sell their homes at 3-18 percent higher than would-be "winners";
- take much longer to sell their houses.

Their suggested explanation is myopic loss aversion:

- Loss aversion combined with a mental-accounting assumption that you experience gain-loss utility from (nominal) financial gains and losses upon selling your house.

Based on Pope & Schweitzer (2011)

In most professional golf tournaments, players play 72 holes, and the order of finish (and hence earnings) is entirely determined by the total number of shots taken over those 72 holes (with the lowest total being the best).

- In addition, each individual hole has a suggested score (*par*). But this suggested score is completely irrelevant to the order of finish and earnings in the tournament.

Question: Suppose I face a 10-foot putt? Should my effort and concentration on this putt, or my strategy on this putt (i.e., being aggressive vs. safe) depend on whether the putt is for "par" vs. whether the putt is for one better than par (a "birdie")?

Pope & Schweitzer (2011) analyze data from 239 PGA Tour tournaments completed between 2004 and 2009.

- Data from all golfers who attempted at least 1000 putts. This is 421 golfers and over 2.5 million putts.
- Data also contain exact location of the ball and the hole (based on video tracking software that was relatively new at the time).

Main Finding

(from Pope & Schweitzer *AER* 2011)

Controlling for the distance of the putt, on average golfers are about 2 percentage points more likely to make a par putt than they are to make a birdie putt.

Note: This finding is robust to many additional controls: controlling for the specific golfer, the specific hole, whether one has had prior putts on the same green, and the direction of the putt.

How to interpret this main finding?

- Under the assumption that golfers care only about their tournament results (and earnings), this behavior is inconsistent with the standard model. Again, whether a putt is for par vs. birdie is irrelevant to tournament results.

Pope & Schweitzer's suggested explanation is loss aversion:

- Loss aversion combined with a mental-accounting assumption that golfers experience gain-loss utility on each hole from performing better or worse than par on that hole.

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Experiment 5 from Kahneman, Knetsch, & Thaler (1990)

- Subjects: 59 students in business statistics class at Simon Fraser University.
- 30 subjects randomly chosen to be *sellers*.
- 29 subjects randomly chosen to be *buyers*.
- Each seller given a coffee mug.
- Each buyer shown a coffee mug.

Then elicit people's reservation values (or reservation prices):

- A buyer's WTP is the *maximum* amount she is willing to pay to obtain the object.
- A seller's WTA is the *minimum* amount she is willing to accept to part with the object.

(If helpful, think about WTA and WTP in terms of indifference.)

Becker, DeGroot, Marschak (*BehSci* 1964) Procedure:

- Participants make lots of decisions for lots of hypothetical prices.
- E.g., "Would you like to keep or sell at price $1?" ... "at price $2?" etc.
- Inform subjects that a price will be randomly selected and their choice for that price will be implemented.

Explain to subjects two true facts:

1. Their choice cannot affect the price (so there is no reason to behave strategically).
2. This implies that the best thing for them to do is to indicate their true preferences.

Reservation prices in Experiment 5:

	Median	**Mean**
Sellers:	\$5.75	\$5.78
Buyers:	\$2.25	\$2.21

Endowment Effect

(Term first used in Thaler, 1980)

- People tend to value an object more highly when they own it than when they do not.

But, qualitatively, standard wealth effects imply that we should expect $WTA > WTP$ in Experiment 5.

> Do you remember what this phrase means?

Experiment 6 from Kahneman, Knetsch, & Thaler (1990)

77 subjects at SFU randomly assigned to three groups:

- Sellers (as before).
- Buyers (as before).
- *Choosers* who indicate for each price whether they want the mug or the money.

Reservation Prices in Experiment 6:

	Median
Sellers:	$7.12
Buyers:	$2.87
Choosers:	$3.12

Conclusion: Because sellers and choosers face the exact same choice and yet reservation values are larger for sellers, standard wealth effects cannot explain the endowment effect.

Knetsch (AER 1989): Endowment Effect with Two Goods

3 groups of subjects (who were present at different times).

- Group 1: Given a coffee mug. Opportunity to trade for a candy bar.
- Group 2: Given a candy bar. Opportunity to trade for a coffee mug.
- Group 3: Choose between a coffee mug & a candy bar.

Results:

	Percent Selecting Mugs
Endowed with a mug:	89%
Endowed with candy:	10%
Choosers (unendowed)	56%

A Simple Model of Loss Aversion & the Endowment Effect

Suppose you consume mugs and money, where

$$(\textbf{Total Utility}) = (\textbf{Mug Utility}) + (\textbf{Money Utility}).$$

Let's also assume linear money utility --- if your consumption of money is $\boldsymbol{m}$, then (Money Utility) $= \boldsymbol{m}$.

$$\Longrightarrow (\textbf{Total Utility}) = (\textbf{Mug Utility}) + \boldsymbol{m}.$$

Suppose your Mug Utility is $\boldsymbol{u(c,r)}$, where $\boldsymbol{c}$ is your mug consumption and $\boldsymbol{r}$ is your mug reference point.

- $r = 0 \iff$ unendowed (buyers & choosers)
- $r = 1 \iff$ endowed (sellers)
- $c = 1 \iff$ go home with mug (buy, choose, or keep).
- $c = 0 \iff$ go home without mug (don't buy, don't choose, or sell)

Assume $\boldsymbol{u\left(c,r\right) = w(c) + v(c-r)}$, where

$$w(c) = \mu * c$$

$$\text{and } v(x) = \phi x \quad \text{if } x \geq 0$$

$$\text{else } v(x) = \lambda\phi x \quad \text{if } x < 0.$$

Note: We can represent $u(c, r)$ in a 2x2 grid:

(Presented in class. Rows are $c = 0$, $c = 1$. Columns are $r = 0$, $r = 1$)

Solving the Model

Reservation values for the three types:

Sellers: $P_S = \mu + \lambda\phi$, implying you sell if $p > P_S$

Buyers: $P_B = \mu + \phi$, implying you buy if $p < P_B$

Choosers: $P_C = \mu + \phi$, implying you choose mug if $p < P_C$

Key fact: $\lambda > 1 \implies P_S > P_C = P_B$.

Robustness of the Effect

Some ways to make the endowment effect go away:

1. Use sufficiently similar goods.
2. Alter procedures to weaken the sense of endowment.
3. Trigger certain emotions (e.g., disgust \& sadness).
4. Market experience (?)

Evidence from List (2003)

Knetsch (1989)-style experiments: two goods, subjects randomly endowed with one of them and given the opportunity to switch (after a brief delay/survey).

Experiment 1

Conducted at a sportscard show in Orlando, FL.

- Good A: A Kansas City Royals game ticket stub from the game in which Cal Ripken broke the record for most consecutive games.
- Good B: A certificate commemorating Nolan Ryan's 300th win that was distributed to fans at that game.
- The subjects were dealers & non-dealers, and he separates non-dealers into experienced and inexperienced.

Results

	Percent Who Trade
Dealers:	45%
Experienced Non-Dealers:	46.7%
Inexperienced Non-Dealers:	6.8%

Adapted from Sydnor (2010)

Analyzes data on homeowner's insurance for 50,000 households. He investigates people's choice of deductible.

For each customer, he observes the person's menu and the person's choice --- for instance, he might observe:

Deductible	Premium	Choice
$1000	$505	.
$500	$588	X
$250	$661	.
$100	$771	.

- In the previous example, this person chose to pay an extra \$84 to reduce his deductible from \$1000 to \$500.

Let's translate this into our language:

This person prefers paying a premium of \$588 for a \$500 deductible over paying a premium of \$504 for a \$1000 deductible.

Let p denote the probability of making a claim during the policy term, and for simplicity let's assume that all claims are larger than \$1000.

Then the person's choice reveals:

$$(-\$588, 1-p; -588-500, p) \succeq (-\$504, 1-p; -504-1000, p)$$

Above is one observation. What about averages?

Striking fact: Among those who chose the \$500 deductible (48% of the sample), on average they paid \$99.85 in extra premium to reduce their deductible from \$1000 to \$500. Moreover, the average claim rate in this group is 4.3%, and so (roughly) the expected value of reducing the deductible is \$21.50 (i.e., .043(1000-500)).

Following Rabin (2000), Sydnor demonstrates that EU with reasonable levels of risk aversion cannot explain this behavior.

- For instance, with CRRA utility, initial wealth \$10,000, \$1000 deductible, and \$500 premium, an EU maximizer with a 4.3% claim rate would choose to pay \$99.85 in extra premium to reduce their deductible from \$1000 to \$500 only if $\rho > \mathbf{20.29}$.

In contrast, prospect theory **might** be able to explain it:

NOTE: Kahneman & Tversky (1979) prospect theory cannot.

Prospect theory combined with assumption that the premium is not felt as a loss **can**.

- Köszegi-Rabin loss aversion (our final model in this section) captures this idea.