Choice under Uncertainty (Lecture 1c)

EC404; Spring 2024

Prof. Ben Bushong

Last updated January 23, 2024

Our Outline:

(1) Guiding Principles

(2) Introduction to Reference Dependence

(3) A New Model: Prospect Theory

(4) Loss Aversion

(5) Diminishing Sensitivity

(6) The Value Function

#2: Use Extreme Cases to Clarify Your Thinking

Some concepts are hard to get your head around. It can be easier to think about things in the extreme limit.

- E.g., "risk aversion" will continue to trip you up throughout this section.
- Think about the limit case: a person who is **infinitely** risk averse.
- Now think about the other limit case: a person who is risk neutral.
- In between those lies reality. The **definition** of the term only offers that a person is a teensy tiny itty bitty bit above risk neutrality. How she actually behaves depends on *how* risk averse she is.

Your task: think about limit cases.

Moe: " If you want to signal me, use this bird call."

[Moe whistles like a bird. An eagle swoops down and pecks him on the face.]

"Ow! Not the face!"

[The eagle switches to pecking Moe in the groin.]

"Ooh! Ooh! Okay, the face!

[The eagle switches back.]

"Ooh! Whoa, that actually feels good after the crotch!"

A Simple Truth

In virtually all physiological and psychological reactions, people's responses tend to reflect adaptation, change, and contrast, rather than solely absolute levels of outcomes.

- Feelings (and, just as importantly, choice) are reference-dependent.
- This suggests a modifictation to the models that we use.

> We should consider a modified utility function $u(x; r)$ rather than $u(w + x)$, where r is some reference point or reference level.

We'll explore this idea for the next few lectures. There are deep implications for economics in this simple observation.

Amos Tversky and Daniel Kahneman worked on this in the 1970s. Kahneman won the Nobel Prize in Economics in 2002 for their joint work.

Propect Theory proposes two phases of choice process:

1. Editing
2. Evaluation

We begin our discussion with the former, but our focus today will be on the latter.

Editing Stage

The psychology of the editing stage is straightforward: a person needs to organize & reformulate some complex situation into a simplified problem.

- More concretely: a choice problem is described to you, and then you transform it into the lotteries that you will evaluate.

Some Examples:

- **Coding:** code outcomes as gains (or losses) relative to some reference point.
- **Cancellation:** discard shared components.
- **Simplification:** rounding off probabilities.
- Eliminating dominated alternatives.

This is an example of the type of thing we won't spend a lot of time on in this course, but it was important to early pioneers.

Evaluation in a Nutshell

Of course, once we face a decision problem we must evaluate it.

Kahneman and Tversky (1979, p. 277) stress that attending to changes from reference points is a basic aspect of human nature:

> Our perceptual apparatus is attuned to the evaluation of changes or differences rather than to the evaluation of absolute magnitudes ... The same principle applies to non-sensory attributes such as health, prestige, and wealth.

The two key features of evaluation emphasized by Kahneman and Tversky (1979) and subsequently by many others:

1. Loss Aversion
2. Diminishing Sensitivity

Loss Aversion: A "Definition"

(Note: this is one of the few times where you can just rely on your intuitive response to the terms. It means exactly what you think it means.)

People dislike losses more than they like same-sized gains.

- Vast majority of people turn down 50/50 lose $500, gain $550 bet
- As highlighted last time, this is *not* due to curvature in utility function.
- Not discussed last time: the strongest such aversion appears to involve mixes of gains and losses.

Loss aversion is an absolutely central component of prospect theory.

It's existence or importance remains a source of scholarly debate.

> Note that I say scholarly. I suspect the average person would immediately agree with the assertion that losses > gains.

My View of Loss Aversion

It is central in a number of everyday activities:

- Moral considerations (e.g., Hippocratic Oath)
- "Endowment Effect" or "Status Quo Bias" in financial trades
- "Disposition Effects", in investments and houses
- Aversion to (nominal) wage and consumption declines
- Income-targeting

Diminishing Sensitivity: A "Definition"

In the following pairs, which "feel" like a bigger difference?

Option A	Option B
visually 101 ft. away vs. 100 ft. away	1 ft. v. 0 ft.
carrying a suitcase 21 v. 20 blocks	2 v. 1 block
gain 100 days from now v. 101 days	gain 0 days v. 1 day
19% chance v. 18% chance	1% chance v. 0%
gaining $101 v. gaining $100	gaining $1 v. gaining $0
losing $101 v. losing $100	losing $1 v. losing $0
losing $101 v. losing $100	losing $2 v. losing $1

Diminishing Sensitivity: A Better "Definition"

People pay less attention to incremental differences when changes are further away from the reference point.

- Prefer $420 for sure or 50/50 chance at $900?
- Prefer losing $420 for sure or 50/50 chance to lose $900?

Reflects big and general fact about human psychology:

We most often think in terms of proportions rather than absolutes.

A person evaluates a prospect $(x, p; y, q)$ according to:

$$V(x, p; y, q) = \pi(p)\, v(x) + \pi(q)\, v(y)$$

$$U(x, p; y, q) = pu(w + x) + qu(w + y) + (1 - p - q)\, u(w)$$

What's new?

$\pi(\cdot)$ is the **probability-weighting function**.

$v(\cdot)$ is the **value function**.

Put in a different notation, a person evaluates a prospect $(x_1, p_1; \ldots; x_n, p_n)$ according to:

$$V(x_1, p_1; \ldots; x_n, p_n) \quad = \sum_{i=1}^{N} \pi(p_i)\, v(x_i).$$

Contrast this with the Expected Utility definition

$$EU(x_1, p_1; \ldots; x_n, p_n) = \sum_{i=1}^{N} p_i u(w + x_i)$$

and that of Expected Value

$$EV(x_1, p_1; \ldots; x_n, p_n) = \sum_{i=1}^{N} p_i x_i$$

Breaking Down the Components of the Theory

Three key features of the value function $v(\cdot)$:

(1) The carriers of value are **changes** in wealth. Thus: $v(0) = 0$.

- Thus: $v(0) = 0$.
- Implicit in this assumption is that the reference point is current wealth.
- There are lots of examples where this is a bad assumption.

(2) *Diminishing sensitivity* to the magnitude of changes.

- Formally: $v'(x) > 0$ for all x, and $v''(x) < 0$ for $x > 0$, while $v''(x) > 0$ for $x < 0$.

(3) *Loss aversion*, or losses loom larger than gains.

- Sloppy formality: $v(x) < v(-x)$ for all $x > 0$.
- Formally: $v(x) + v(-x) < v(y) + v(-y)$ for all $x > y$.

These assumptions lead to the following visual form of the value function:

(see board; or just google it if you're not in class.)

A functional form that's often used:

$$v(x) = \{x^{\alpha} \quad \text{if} \quad x \geq 0$$

$$\lambda(x)^{\beta} \quad \text{if} \quad x \leq 0$$

An even easier functional form that we will mostly use eliminates the exponents.

Note: This second functional form removes diminishing sensitivity and isolates the effect of loss aversion on decision-making. In lots of settings this will greatly simplify the problem while leaving the fun stuff intact.

Implications (and non-Implications)

If a person maximizes her preferences meeting the assumptions above, she...

(i) ...will turn down any 50/50 lose $X, gain $X bets.

-Implication (i) is implied by Loss Aversion.

- Non-Implication (i). "...is necessarily averse to all fair bets."
- The assumptions do *not* guarantee a person will turn down all fair bets.

(ii) ...is risk averse among bets involving only gains.

- Implication 2 is implied directly by Diminishing Sensitivity.

(iii)... is risk-*loving* among bets involving only losses.

Implication (iii) is also implied directly by Diminishing Sensitivity.

(iv) ... is " first-order risk-averse."

- Implication (iv) requires important additional assumption.

Let x be a **random variable** with distribution $F(x)$.

Let $\mathbb{E}(x)$ denote the expectation of x and σ_x^2 the variance.

Consider the lottery $k + x$ as the lottery that pays k plus the realization of the random variable x.

Claim: Let $\mathbb{E}(x) = 0$ and consider an expected utility maximizer. Suppose $t > 0$ such that $-\pi \sim t \cdot x + k$. Then

$$\pi \approx \frac{-t^2\sigma_x^2}{2} \frac{u''(w+k)}{u'(w+k)}$$

Put another way: the "risk premium" decreases at rate t^2, while the "size" of the risk decreases at rate t.

First-Order Risk Aversion

MICHIGAN STATE UNIVERSITY

Thus for small risks, a person must be almost *risk neutral*: the "premium" required to take on that risk would go to zero as the size of the risk goes to zero.

Assumption: A decision maker is *first-order risk averse* if for prospect theory value function $v(\cdot)$:

$$\lim_{x \to 0} \frac{v'(-x)}{v'(x)} \equiv L > 1$$

when approached from the $x > 0$ direction.

We will carry this assumption through many of our functional forms.

Recall:

$$V(x, p; y, q) \quad = \quad \pi(p)\, v(x) \quad + \quad \pi(q)\, v(y)$$

We turn to key features of the probability-weighting function $\pi(\cdot)$:

[EU theory says $\pi(p) = p$.]

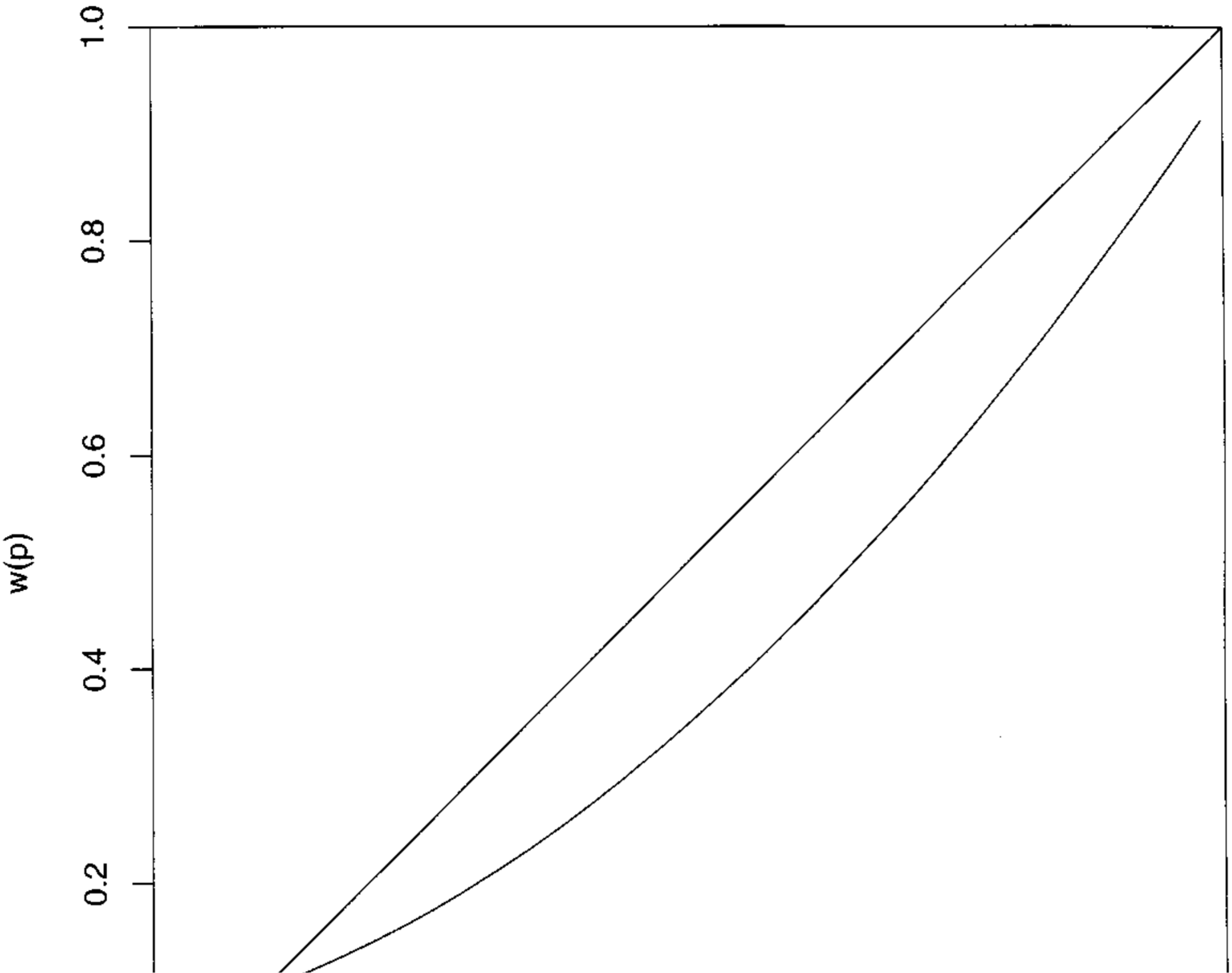
Natural assumptions:

- $\pi(0) = 0$, $\pi(1) = 1$, and π is increasing.
- Subcertainty: $\pi(p) + \pi(1 - p) < 1$.
- Subproportionality:

$$\frac{\pi(pq)}{\pi(p)} \leq \frac{\pi(pqr)}{\pi(pr)}$$

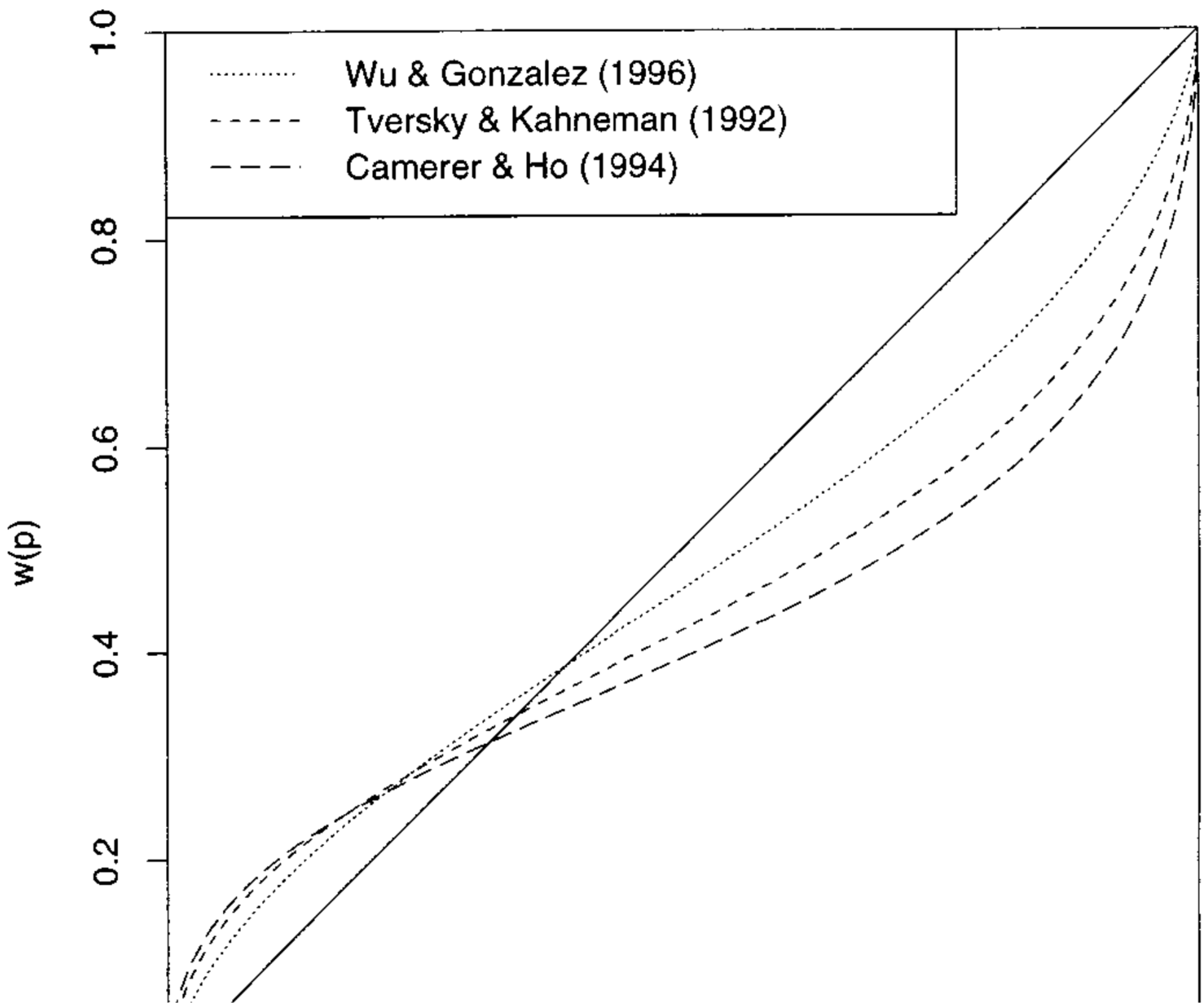
  for $p, q, r \in (0, 1)$.

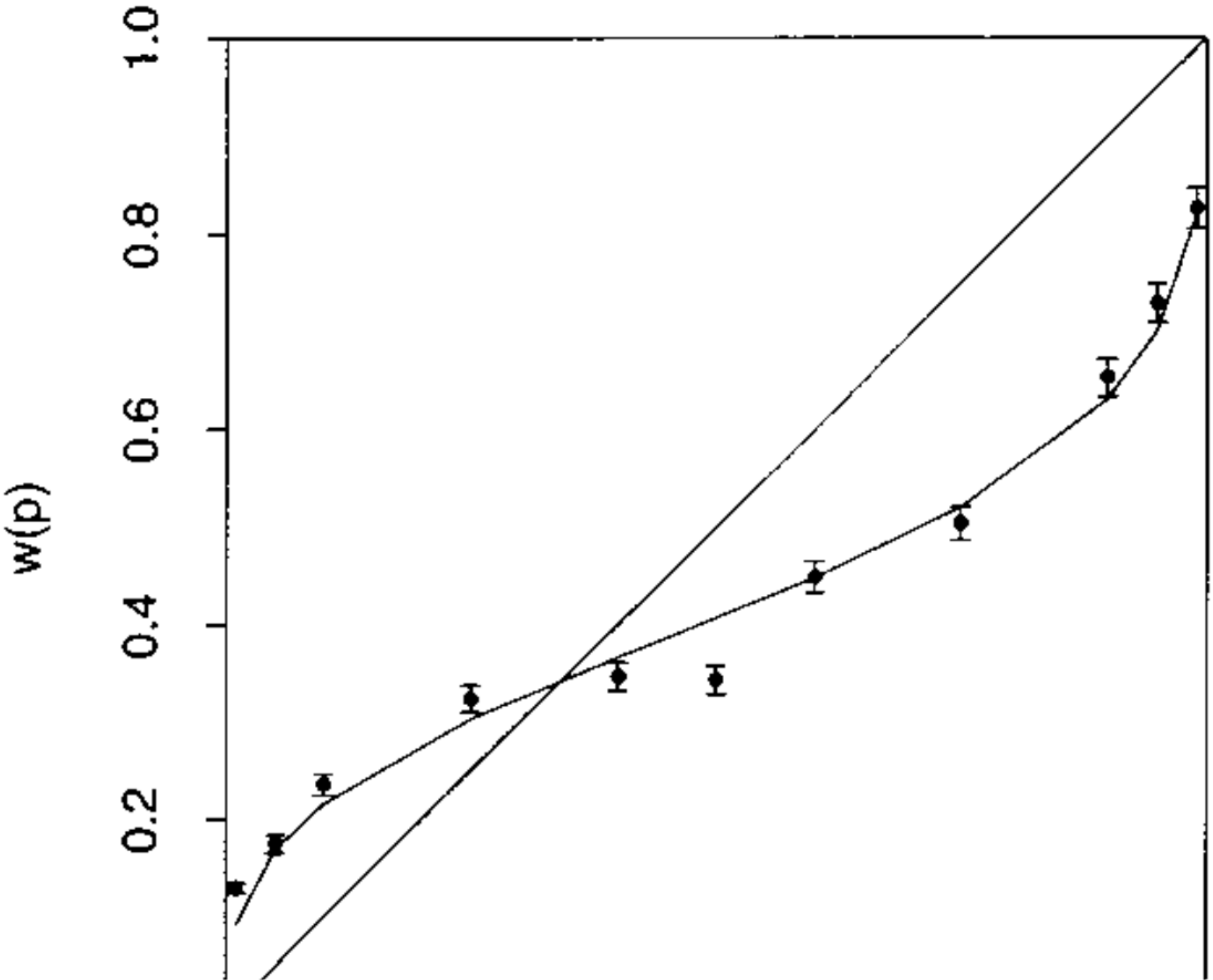
- For small p, $\pi(p) > p$.


1.0
0.8
0.6
0.4
0.2
w(p)

Probability-Weighting Function

MICHIGAN STATE UNIVERSITY




w(p)
1.0
0.8
0.6
0.4
0.2