Problem Set 1: Snap

[Due in class on Monday]

Question 1:

Consider the following choice:

$$\mathbf{x} \equiv (\$1000, 0.7; -\$1000, 0.3) \quad \text{vs.} \quad \mathbf{y} \equiv \left(\$600, \frac{1}{3}; \$300, \frac{1}{2}; \$0, \frac{1}{6}\right)$$

(a) Ting is an EV maximizer with initial wealth \$10,000. Which does she choose?

(b) Grace and Sunflower are EU maximizers with initial wealth \$10,000.

(i) If Grace has a CRRA utility function with $\rho = 1/2$, which does she choose?

(ii) If Sunflower has a CRRA utility function with $\rho = 2$, which does she choose?

Question 2:

Suppose that Sonya has initial wealth \$20,000 that she plans to invest for a year and then consume it. There are two assets in which she can invest. First, there is a risk-free asset that yields a certain return of 3%. Second, there is a risky asset that yields a return of 45% with probability 1/3 and a return of -15% with probability 2/3.

If Sonya is an expected-utility maximizer with utility function $u(x) = \ln x$, how much would she invest in each asset? (Note: This requires calculus.)

Question 3:

Consider the following three choice situations:

Choice (i): $(\ \$2500, 1/2\ ;\ \$0, 1/2\ )$ vs. $(\ \$1300, 1\ )$

Choice (ii): $(\ -\$6000, 1/4\ ;\ -\$1000, 3/4\ )$ vs. $(\ -\$2300, 1\ )$

Choice (iii): $(\ \$600, 1/4\ ;\ \$400, 1/4\ ;\ \$200, 1/4\ ;\ \$0, 1/4\ )$ vs. $(\ \$400, 1/2\ ;\ \$200.1/2\ )$

(a) Suppose Josh is a risk-averse expected utility maximizer. For each choice, can we determine which option Josh will choose, or do we need more information?

(b) For each choice, as Josh becomes more risk averse, does he become more prone to choose the first or second option?

(c) Now suppose Josh has a CRRA utility function. For each choice, as Josh's wealth becomes larger, does he become more prone to choose the first or second option?

(Note: You should be able to intuit an answer to part **(c)** — no math is needed.)

Question 4: (Demand for Insurance)

Suppose you are a risk-averse expected utility maximizer with utility function $u(x)$. You have initial wealth \$20,000, but before you consume it, you are subject to the following health risk (these events are mutually exclusive):

Required Medical Payment	Probability
\$100	20%
\$500	8%
\$1500	2%
\$10,000	1%

Now suppose that an insurance agent offers to sell you health insurance with 10% coinsurance — that is, for any medical payment that you require, you must pay for 10% of the payment and the insurance company pays the rest. The price of this insurance is p.

(a) If you do not buy the insurance, what lottery do you face? If you buy the insurance, what lottery do you face?

(b) Let p^* denote your willingness to pay for the insurance — so that you prefer to buy the insurance if $p < p^*$ and you prefer not to buy the insurance if $p > p^*$. Provide an equation from which you could derive p^*.

(c) If you are risk-neutral, what can we say about your p^*? If you are risk-averse, what can we say about your p^*? As you become more risk averse, what happens to your p^*? Briefly explain your answers.

(d) If the coinsurance rate goes up to 15%, what happens to your p^*? Briefly explain your answer.

Question 5:

Consider the following choice (where $p \in [0,1]$, $q \in [0,1]$, and $p+q \leq 1$):

$$\mathbf{x} \equiv (\ \$600, p\ ;\ \$400, q\ ;\ \$0, 1-p-q\ ) \quad \text{vs.} \quad \mathbf{y} \equiv \left(\ \$500, \frac{1}{3}\ ;\ \$300, \frac{1}{2}\ ;\ \$0, \frac{1}{6}\ \right)$$

(a) For what values of p and q does lottery **x** dominate lottery **y**?

(b) For what values of p and q does lottery **y** dominate lottery **x**?

Question 6:

Suppose you face a potential loss L that will occur with probability q. An insurance agent offers to let you buy partial insurance, wherein you can pay αp to insure against proportion α of this loss. But you are restricted to choosing $\alpha \in [0,1]$.

(a) Suppose the insurance is actuarially unfair ($p > qL$).

(i) If you are risk-averse, what can we conclude about the α that you might choose?

(ii) If you are risk-loving, what can we conclude about the α that you might choose?

(b) Suppose the insurance is actuarially overfair ($p < qL$).

(i) If you are risk-averse, what can we conclude about the α that you might choose?

(ii) If you are risk-loving, what can we conclude about the α that you might choose?

Note: Question 1 should be answered using only intuition/logic — no calculus is needed.

Question 7:

Suppose a person chooses Lottery A over Lottery B, but also chooses Lottery D over Lottery C, where:

Lottery A: $(1000, 1)$

Lottery B: $(2000, .2; 1000, .7; 0, .1)$

Lottery C: $(2000, .2; 0, .8)$

Lottery D: $(1000, .3; 0, .7)$

(a) Does this person's behavior violate expected utility (without any restrictions on u)?

(b) Does this person's behavior violate expected utility with more is better?

(c) Does this person's behavior violate expected utility with risk aversion?

Explain your answers.

Question 8:

Suppose that Liam evaluates gambles according to prospect theory with $\pi(p) = p$ and a value function

$$v(x) = \begin{cases} x^{0.88} & \text{if } x \geq 0 \\ -2.25 * (-x)^{0.88} & \text{if } x \leq 0. \end{cases}$$

Liam owns an asset that yields a lottery $(\$1000, \frac{1}{3}; \$100, \frac{1}{2}; -\$1000, \frac{1}{6})$. If you offer to purchase this asset from Liam for an amount $\$z$, how large would $\$z$ need to be for Liam to accept your offer?

Question 9:

Suppose Martin is a risk-averse expected utility maximizer. In contrast, Roberto evaluates gambles according to prospect theory with $\pi(p) = p$ and a value function that has the three properties suggested by Kahneman & Tversky.

Consider the following four choice situations:

Choice (i): $(\ \$8000, \frac{1}{8}\ ;\ \$2000, \frac{7}{8}\ )$ vs. $(\ \$2800, 1\ )$

Choice (ii): $(\ -\$800, \frac{2}{5}\ ;\ -\$400, \frac{3}{5}\ )$ vs. $(\ -\$550, 1\ )$

Choice (iii): $(\ -\$1700, \frac{1}{2}\ ;\ \$0, \frac{1}{2}\ )$ vs. $(\ -\$875, 1\ )$

Choice (iv): $(\ -\$200, \frac{4}{5}\ ;\ \$800, \frac{1}{5}\ )$ vs. $(\ \$0, 1\ )$

For each choice, describe for both Martin and Roberto whether we can determine which option they will choose or whether we need more information.

Question 10:

Suppose that Jennifer evaluates gambles according to prospect theory with $\pi(p) = p$ and a value function that has the three properties suggested by Kahneman & Tversky.

(a) If Jennifer chooses lottery $(\$800, .9; \$0, .1)$ over lottery $(\$1400, .5; \$0, .5)$, could she also choose lottery $(\$1400, .25; \$0, .75)$ over lottery $(\$800, .45; \$0, .55)$? Could prospect theory with $\pi(p) \neq p$ explain this pattern of choices? If so, how? If not, why not?

(b) If Jennifer faces a 2% chance of incurring a loss of \$20,000, would she be willing to purchase full insurance at a premium of \$400? Does prospect theory with $\pi(p) \neq p$ make a prediction for how a person should behave? If so, what is it? If not, why not?

(c) If Jennifer faces a 60% chance of incurring a loss of \$2000, would she be willing to purchase full insurance at a premium of \$1200? Does prospect theory with $\pi(p) \neq p$ make a prediction for how a person should behave? If so, what is it? If not, why not?

Question 11:

Consider the bet ($\$400, \frac{1}{3}$; $-\$Y, \frac{2}{3}$).

(a) Suppose that Heidi is an expected utility maximizer with $u(x) = \ln x$, and her initial wealth is \$12000.

(i) For what values of Y does Heidi reject a single play of the bet?

(ii) For what values of Y does Heidi accept two independent plays of the bet?

(iii) Is it possible for Heidi to reject the single bet but accept the aggregate bet?

Note: For part (a), report your answers to 3 decimal points.

(b) Suppose that Bruce also has initial wealth \$12000, but he evaluates gambles according to prospect theory with $\pi(p) = p$ and

$$v(x) = \begin{cases} x & \text{if } x \geq 0 \\ 2.5x & \text{if } x \leq 0. \end{cases}$$

(i) For what values of Y does Bruce reject a single play of the bet?

(ii) For what values of Y does Bruce accept two independent plays of the bet?

(iii) Is it possible for Bruce to reject the single bet but accept the aggregate bet?

Question 12

(a) Suppose that Johnny is an expected utility maximizer with $u(x) = -e^{-0.001x}$, and has initial wealth is $75,000. Derive how Johnny feels about the following bets:

(i) Johnny will accept $(-100, \frac{1}{2}; X, \frac{1}{2})$ if and only if $X >$????

(ii) Johnny will accept $(-200, \frac{1}{2}; X, \frac{1}{2})$ if and only if $X >$????

(iii) Johnny will accept $(-500, \frac{1}{2}; X, \frac{1}{2})$ if and only if $X >$????

(iv) Johnny will accept $(-750, \frac{1}{2}; X, \frac{1}{2})$ if and only if $X >$????

(b) Like Johnny, Tommy has initial wealth $75,000. But unlike Johnny, Tommy evaluates gambles according to prospect theory with $\pi(p) = p$ and

$$v(x) = \begin{cases} x & \text{if } x \geq 0 \\ \lambda x & \text{if } x \leq 0. \end{cases}$$

If we observe that Tommy accepts $(-100, \frac{1}{2}; X, \frac{1}{2})$ if and only if $X > 210$, what can we conclude about Tommy's λ?

(c) Suppose that Tommy has the λ that you found in part **(b)**, and derive how he feels about the following bets:

(i) Tommy will accept $(-200, \frac{1}{2}; X, \frac{1}{2})$ if and only if $X >$????

(ii) Tommy will accept $(-500, \frac{1}{2}; X, \frac{1}{2})$ if and only if $X >$????

(iii) Tommy will accept $(-750, \frac{1}{2}; X, \frac{1}{2})$ if and only if $X >$????

Question 13

Suppose that you have \$1000 to invest, and you can invest it in stocks or bonds. Each month, bonds yield a certain return of 0.8%. Each month, stocks yield a risky return of 1.8% with probability 0.7 and -1.1% with probability 0.3. Assume returns are independent across months.

You choose your portfolio as suggested by Benartzi & Thaler. Let x be the change in your portfolio's value between now and the next time you evaluate your portfolio. Of course x will be risky — that is, your choice will generate a lottery over possible outcomes for x. For any lottery $(x_1, p_1; ...; x_N, p_N)$, you evaluate this lottery according to prospective utility

$$\sum_{i=1}^{N} p_i \, v(x_i)$$

where

$$v(x_i) = \begin{cases} x_i & \text{if } x_i \geq 0 \\ 2.25x_i & \text{if } x_i \leq 0. \end{cases}$$

(a) Suppose you plan to evaluate your portfolio after one month. If you invest in all bonds, what is the lottery over x? If you invest in all stocks, what is the lottery over x? Which do you prefer, all bonds or all stocks?

(b) Suppose you plan to evaluate your portfolio after two months. If you invest in all bonds, what is the lottery over x? If you invest in all stocks, what is the lottery over x? Which do you prefer, all bonds or all stocks?

(c) How does your preference for stocks vs. bonds depend on your evaluation horizon? Discuss the significance of your answer for the equity premium puzzle.

(d) Suppose you plan to evaluate your portfolio after two months, and suppose further that you consider splitting your \$1000 evenly between stocks and bonds. How do you feel about this allocation relative to all bonds or all stocks?