Problem Set 6a

EC404 Spring 2024

[Due, with Part b, on April 18th]

Question 1:

This question asks you to extend the medical-test example from class. Suppose there is a medical condition that afflicts 20% of the population. There is a test for this condition, and the reliability of this test can be characterized as follows:

- false-positive rate = 10% — for those who do not have the condition, 10% will test positive.
- false-negative rate = 5% — for those who have the condition, 5% will test negative.

(a) If you receive a positive test result, what is the likelihood that you have the condition? If you receive a negative test result, what is the likelihood that you do not have the condition?

(b) Repeat part (a), except now assume that the false-positive rate is 5% and the false-negative rate is 10%.

Question 2:

Suppose there are two types of dice, red dice and blue dice. Each red die has 4 H's and 2 M's, whereas each blue die has 2 H's and 4 M's. The proportion of all dice that are red is 80%.

For each of the scenarios below, discuss whether the person's intuitive judgment is consistent with (i) base-rate neglect, (ii) over-inference from small samples, and (iii) conservatism.

(a) A person is told that a die was rolled three times and came up MMH. When asked the likelihood that the die is red, the person responds $1/2$.

(b) A person is told that a die was rolled three times and came up HMH. When asked the likelihood that the die is red, the person responds $9/10$.

(c) A person is told that a die was rolled ten times and came up $MMHHMHMHHH$. When asked the likelihood that the die is red, the person responds $17/20$.

Question 3:

Suppose there are two types of coins, heads-biased coins and tails-biased coins. A heads-biased coin has a $3/4$ probability of a heads, while a tails-biased coin has a $1/4$ probability of heads. The proportion of all coins that are heads-biased is $1/7$.

Suppose that we flip a coin twice and it comes up HH.

(a) For a standard Bayesian information processor:

(i) What is the person's posterior probability that the coin is heads-biased?

(ii) What is the person's forecast for a third flip being H?

(b) For an ($N = 8$)-Freddy (as defined in class):

(i) What is the person's posterior probability that the coin is heads-biased?

(ii) What is the person's forecast for a third flip being H?

(c) Repeat parts (a) and (b) when the proportion of all coins that are heads-biased is $6/7$.

(d) How do Freddy's forecasts compare to a Bayesian's forecasts? Provide some intuition for your conclusions.