Problem Set 2

[Due in class on Tuesday, February 6.]

Question 1:

Suppose you face a potential loss L that will occur with probability q. An insurance agent offers to let you buy partial insurance, wherein you can pay αp to insure against proportion α of this loss. But you are restricted to choosing $\alpha \in [0,1]$.

(a) Suppose the insurance is actuarially unfair ($p > qL$).

(i) If you are risk-averse, what can we conclude about the α that you might choose?

(ii) If you are risk-loving, what can we conclude about the α that you might choose?

(b) Suppose the insurance is actuarially overfair ($p < qL$).

(i) If you are risk-averse, what can we conclude about the α that you might choose?

(ii) If you are risk-loving, what can we conclude about the α that you might choose?

Note: Question 1 should be answered using only intuition/logic — no calculus is needed.

Question 2:

Suppose a person chooses Lottery A over Lottery B, but also chooses Lottery D over Lottery C, where:

Lottery A: $(1000, 1)$

Lottery B: $(2000, .2; 1000, .7; 0, .1)$

Lottery C: $(2000, .2; 0, .8)$

Lottery D: $(1000, .3; 0, .7)$

(a) Does this person's behavior violate expected utility (without any restrictions on u)?

(b) Does this person's behavior violate expected utility with more is better?

(c) Does this person's behavior violate expected utility with risk aversion?

Explain your answers.

Question 3:

Suppose that Liam evaluates gambles according to prospect theory with $\pi(p) = p$ and a value function

$$v(x) = \begin{cases} x^{0.88} & \text{if } x \geq 0 \\ -2.25 * (-x)^{0.88} & \text{if } x \leq 0. \end{cases}$$

Liam owns an asset that yields a lottery $(\$1000, \frac{1}{3}; \$100, \frac{1}{2}; -\$1000, \frac{1}{6})$. If you offer to purchase this asset from Liam for an amount $\$z$, how large would $\$z$ need to be for Liam to accept your offer?

Question 4:

Suppose Martin is a risk-averse expected utility maximizer. In contrast, Roberto evaluates gambles according to prospect theory with $\pi(p) = p$ and a value function that has the three properties suggested by Kahneman & Tversky.

Consider the following four choice situations:

Choice (i): $(\ \$8000, \frac{1}{8}\ ;\ \$2000, \frac{7}{8}\ )$ vs. $(\ \$2800, 1\ )$

Choice (ii): $(\ -\$800, \frac{2}{5}\ ;\ -\$400, \frac{3}{5}\ )$ vs. $(\ -\$550, 1\ )$

Choice (iii): $(\ -\$1700, \frac{1}{2}\ ;\ \$0, \frac{1}{2}\ )$ vs. $(\ -\$875, 1\ )$

Choice (iv): $(\ -\$200, \frac{4}{5}\ ;\ \$800, \frac{1}{5}\ )$ vs. $(\ \$0, 1\ )$

For each choice, describe for both Martin and Roberto whether we can determine which option they will choose or whether we need more information.

Question 5:

Suppose that Jennifer evaluates gambles according to prospect theory with $\pi(p) = p$ and a value function that has the three properties suggested by Kahneman & Tversky.

(a) If Jennifer chooses lottery $(\$800, .9; \$0, .1)$ over lottery $(\$1400, .5; \$0, .5)$, could she also choose lottery $(\$1400, .25; \$0, .75)$ over lottery $(\$800, .45; \$0, .55)$? Could prospect theory with $\pi(p) \neq p$ explain this pattern of choices? If so, how? If not, why not?

(b) If Jennifer faces a 2% chance of incurring a loss of \$20,000, would she be willing to purchase full insurance at a premium of \$400? Does prospect theory with $\pi(p) \neq p$ make a prediction for how a person should behave? If so, what is it? If not, why not?

(c) If Jennifer faces a 60% chance of incurring a loss of \$2000, would she be willing to purchase full insurance at a premium of \$1200? Does prospect theory with $\pi(p) \neq p$ make a prediction for how a person should behave? If so, what is it? If not, why not?

Question 6:

Consider the bet ($\$400, \frac{1}{3}$; $-\$Y, \frac{2}{3}$).

(a) Suppose that Heidi is an expected utility maximizer with $u(x) = \ln x$, and her initial wealth is $12000.

(i) For what values of Y does Heidi reject a single play of the bet?

(ii) For what values of Y does Heidi accept two independent plays of the bet?

(iii) Is it possible for Heidi to reject the single bet but accept the aggregate bet?

Note: For part (a), report your answers to 3 decimal points.

(b) Suppose that Bruce also has initial wealth $12000, but he evaluates gambles according to prospect theory with $\pi(p) = p$ and

$$v(x) = \begin{cases} x & \text{if } x \geq 0 \\ 2.5x & \text{if } x \leq 0. \end{cases}$$

(i) For what values of Y does Bruce reject a single play of the bet?

(ii) For what values of Y does Bruce accept two independent plays of the bet?

(iii) Is it possible for Bruce to reject the single bet but accept the aggregate bet?

Question 7

(a) Suppose that Johnny is an expected utility maximizer with $u(x) = -e^{-0.001x}$, and has initial wealth is \$75,000. Derive how Johnny feels about the following bets:

(i) Johnny will accept $(-100, \frac{1}{2}; X, \frac{1}{2})$ if and only if $X >$????

(ii) Johnny will accept $(-200, \frac{1}{2}; X, \frac{1}{2})$ if and only if $X >$????

(iii) Johnny will accept $(-500, \frac{1}{2}; X, \frac{1}{2})$ if and only if $X >$????

(iv) Johnny will accept $(-750, \frac{1}{2}; X, \frac{1}{2})$ if and only if $X >$????

(b) Like Johnny, Tommy has initial wealth \$75,000. But unlike Johnny, Tommy evaluates gambles according to prospect theory with $\pi(p) = p$ and

$$v(x) = \begin{cases} x & \text{if } x \geq 0 \\ \lambda x & \text{if } x \leq 0. \end{cases}$$

If we observe that Tommy accepts $(-100, \frac{1}{2}; X, \frac{1}{2})$ if and only if $X > 210$, what can we conclude about Tommy's λ?

(c) Suppose that Tommy has the λ that you found in part **(b)**, and derive how he feels about the following bets:

(i) Tommy will accept $(-200, \frac{1}{2}; X, \frac{1}{2})$ if and only if $X >$????

(ii) Tommy will accept $(-500, \frac{1}{2}; X, \frac{1}{2})$ if and only if $X >$????

(iii) Tommy will accept $(-750, \frac{1}{2}; X, \frac{1}{2})$ if and only if $X >$????

Question 8

Suppose that you have \$1000 to invest, and you can invest it in stocks or bonds. Each month, bonds yield a certain return of 0.8%. Each month, stocks yield a risky return of 1.8% with probability 0.7 and -1.1% with probability 0.3. Assume returns are independent across months.

You choose your portfolio as suggested by Benartzi & Thaler. Let x be the change in your portfolio's value between now and the next time you evaluate your portfolio. Of course x will be risky — that is, your choice will generate a lottery over possible outcomes for x. For any lottery $(x_1, p_1; ...; x_N, p_N)$, you evaluate this lottery according to prospective utility

$$\sum_{i=1}^{N} p_i \, v(x_i)$$

where

$$v(x_i) = \begin{cases} x_i & \text{if } x_i \geq 0 \\ 2.25x_i & \text{if } x_i \leq 0. \end{cases}$$

(a) Suppose you plan to evaluate your portfolio after one month. If you invest in all bonds, what is the lottery over x? If you invest in all stocks, what is the lottery over x? Which do you prefer, all bonds or all stocks?

(b) Suppose you plan to evaluate your portfolio after two months. If you invest in all bonds, what is the lottery over x? If you invest in all stocks, what is the lottery over x? Which do you prefer, all bonds or all stocks?

(c) How does your preference for stocks vs. bonds depend on your evaluation horizon? Discuss the significance of your answer for the equity premium puzzle.

(d) Suppose you plan to evaluate your portfolio after two months, and suppose further that you consider splitting your \$1000 evenly between stocks and bonds. How do you feel about this allocation relative to all bonds or all stocks?

Question 9

In class, we developed a simple model with mug utility and money utility, and we derived implications for the reservation values of buyers, sellers, and choosers in endowment-effect experiments. This question asks you to think through several variants of that model. For all parts, assume that the person has wealth $w = \$15,000$, and that $(\text{Total Utility}) = (\text{Mug Utility}) + (\text{Money Utility})$.

(a) Let's begin with the case studied in class: Suppose that money utility is $u_m(m) = m$, and that mug utility is $u(c,r) = \mu c + v(c-r)$ where

$$v(x) = \begin{cases} \eta_{\text{mug}} * x & \text{if } x \geq 0 \\ \lambda_{\text{mug}} * \eta_{\text{mug}} * x & \text{if } x \leq 0. \end{cases}$$

(i) Derive the reservation values for buyers, sellers, and choosers as a function of μ, η_{mug}, and λ_{mug}.

(ii) Discuss how and why the three types differ. If it helps, consider the special case when $\mu = 3$, $\eta_{\text{mug}} = 0.5$ and $\lambda_{\text{mug}} = 5$.

(b) Now, let's introduce loss aversion over money: Suppose that mug utility is as in part **(a)**, but now suppose that money utility is $u_m(m, r_m) = m + v_m(m - r_m)$ where $r_m = w$ and

$$v_m(x) = \begin{cases} \eta_{\text{money}} * x & \text{if } x \geq 0 \\ \lambda_{\text{money}} * \eta_{\text{money}} * x & \text{if } x \leq 0. \end{cases}$$

(i) Derive the reservation values for buyers, sellers, and choosers as a function of μ, η_{mug}, λ_{mug}, η_{money}, and λ_{money}.

(ii) Discuss how and why the three types differ. If it helps, consider the special case when $\mu = 3$, $\eta_{\text{mug}} = 0.5$, $\lambda_{\text{mug}} = 5$, $\eta_{\text{money}} = 0.3$, and $\lambda_{\text{money}} = 4$.

(c) Next, instead of assuming loss aversion over money, let's assume diminishing marginal utility from money: Suppose again that mug utility is as in part **(a)**, but now suppose that money utility is $u_m(m) = 15,000 * \ln m$. To simplify things, assume that $\mu = 3$, $\eta_{\text{mug}} = 0.5$, $\lambda_{\text{mug}} = 5$.

(i) Derive the reservation values for buyers, sellers, and choosers.

(ii) Discuss how and why the three types differ.

(d) Finally, let's keep diminishing marginal utility from money, but now eliminate loss aversion over mugs: Suppose that mug utility is $u(c) = 3.5c$, and that money utility is $u_m(m) = 15,000 * \ln m$.

(i) Derive the reservation values for buyers, sellers, and choosers.

(ii) Discuss how and why the three types differ.

Note: For parts (c) & (d), report your answers to 4 decimal points.

Question 10: **[optional, no extra credit, nothing. Just another question. Don't want to do it? Don't! Live your life. Go outside. Breathe in the fresh air. Contemplate the universe. Or, you know, answer the questions.]**

Short-Answer Questions: For each question below, please provide a short, concise answer — your answer only needs to be a few sentences.

(a) Does the reflection effect violate expected utility? Briefly explain your answer.

(b) Briefly explain the difference between diminishing sensitivity and loss aversion.