

Problem Set Martin — Suggested Answers

Question 1:

(a) If the insurance is actuarially unfair, a larger α implies less risk and a smaller expected payoff.

(ai) If you are risk-averse, you like less risk but you dislike a smaller expected payoff. This means that increasing α creates a trade-off for you: As α increases, you face less risk, which is good, but you get a smaller expected payoff, which is bad. Hence, we cannot say exactly what α you would choose. If the insurance is moderately unfair, you'll partially insure – you'll choose an $\alpha \in (0, 1)$. If the insurance is sufficiently unfair, you might not insure at all – you'll choose $\alpha = 0$.

[Advanced and not required: With some extra work, one can show that, starting from full insurance, taking on a little risk has essentially no cost. At the same time, taking on that little risk does increase your expected payoff, which is good. Hence, you'll never fully insure – you'll never choose $\alpha = 1$. For more details on this logic, see me.]

(aii) If you are risk-loving, you dislike less risk and you dislike a smaller expected payoff. This means that increasing α makes you unambiguously worse: As α increases, you face less risk, which is bad, and you get a smaller expected payoff, which is also bad. Hence, we can conclude that you'll buy as little insurance as possible – i.e., you'll choose $\alpha = 0$.

(b) If the insurance is actuarially overfair, a larger α implies less risk and a larger expected payoff.

(bi) Now, if you are risk-averse, increasing α makes you unambiguously better: As α increases, you face less risk, which is good, and you get a larger expected payoff, which is also good. Hence, we can conclude that you'll buy as much insurance as possible – i.e., you'll choose $\alpha = 1$.

(bii) Now, if you are risk-loving, increasing α creates a trade-off for you: As α increases, you face less risk, which is bad, but you get a larger expected payoff, which is good. Hence, we cannot say exactly what α you would choose.

[Advanced and not required: Because of the convexity associated with risk-loving preferences and the fact that the lottery is linear in α , in fact one can show that a risk-loving person would

always choose a corner solution of $\alpha = 0$ or $\alpha = 1$. For more details on this conclusion, see me.]

Question 2:

(a) To check whether this person's behavior violates expected utility, we need to check whether the two inequalities implied by these choices can both be satisfied. Under EU, choosing Lottery A over Lottery B implies

$$u(w + 1000) > .2u(w + 2000) + .7u(w + 1000) + .1u(w)$$

or, algebraically rearranging the above:

$$3u(w + 1000) > .2u(w + 2000) + .1u(w).$$

Under EU, choosing Lottery D over Lottery C implies

$$.3u(w + 1000) + .7u(w) > .2u(w + 2000) + .8u(w)$$

or, algebraically rearranging the above:

$$.3u(w + 1000) > .2u(w + 2000) + .1u(w).$$

Since these inequalities are identical, clearly both can be satisfied, and thus the person's behavior is consistent with expected utility.

(b) If in addition to obeying expected utility a person also thinks more is better, then we know he cannot choose a dominated lottery. Here, there is no dominance in either choice. Hence, the person's behavior is consistent with expected utility with more is better. (It is also implied by the logic above).

(c) Note that Lottery A has less risk than Lottery B, but it also has a lower expected value. Hence, choosing either lottery is consistent with expected utility with risk-aversion. Similarly, Lottery D has less risk than Lottery C, but it also has a lower expected value. Hence, choosing

either lottery is consistent with expected utility with risk-aversion. Overall, then, the person's behavior is consistent with expected utility with risk aversion. (It is also implied by the logic above).

Question 3:

The goal here is to find the $\bar{z}$ such that

$$(\bar{z}, 1) \sim (1000, \frac{1}{3}; 100, \frac{1}{2}; -1000, \frac{1}{6}).$$

Applying prospect theory with $\pi(p) = p$, this indifference point implies

$$\begin{aligned} v(\bar{z}) &= \frac{1}{3}v(1000) + \frac{1}{2}v(100) + \frac{1}{6}v(-1000) \\ \text{or } v(\bar{z}) &= \frac{1}{3}(1000)^{.88} + \frac{1}{2}(100)^{.88} + \frac{1}{6}(-2.25)(1000)^{.88} \\ \text{or } v(\bar{z}) &= 10.5838 \end{aligned}$$

Because $v(\bar{z}) > 0$ we must have $\bar{z} > 0$, and thus

$$\bar{z}^{.88} = 10.5838 \quad \text{or} \quad \bar{z} = \$14.60.$$

Hence, Liam would accept your offer as long as $z > \$14.60$.

Question 4:

Similar to a question on Problem Set 1, because Martin is a risk-averse expected utility maximizer, he has two concerns: he dislikes risk, and he likes expected value.

Roberto evaluates gambles according to prospect theory, and so things are slightly different. *When evaluating lotteries that involve no possible losses*, because the value function is concave in the gain domain (and since like EU he has $\pi(p) = p$), Roberto behaves like a risk-averse expected utility maximizer — in particular, Roberto will have two concerns: he dislikes risk, and he likes expected value. However, *when evaluating lotteries that involve no possible gains*, because the value function is convex in the loss domain, Roberto behaves like a **risk-loving** expected utility maximizer — in particular, Roberto will have two concerns: he likes risk, and he likes expected value.

Applying these conclusions to choices (i) thru (iii):

(i) These lotteries involve no possible losses, so Martin and Roberto are both risk-averse. Because the expected value of the risky lottery (2750) is smaller than the expected value of the riskless lottery (2800), we can conclude that both Martin and Roberto will choose the riskless lottery.

(ii) These lotteries involve no possible gains, so Martin is risk-averse while Roberto is risk-loving. Because the expected value of the risky lottery (−560) is smaller than the expected value of the riskless lottery (−550), we can conclude that Martin will choose the riskless lottery. For Roberto, we cannot determine which option he will choose without more information — it depends on how risk-loving Roberto is in the loss domain.

(iii) Again, these lotteries involve no possible gains, so Martin is risk-averse while Roberto is risk-loving. Because the expected value of the risky lottery (−850) is larger than the expected value of the riskless lottery (−875), we can conclude that Roberto will choose the risky lottery. For Martin, we cannot determine which option he will choose without more information — it depends on how risk-averse he is.

(iv) For Martin, it's easy: these lotteries have the same expected value (0), and so Martin will choose the riskless lottery. For Roberto, we must think a little harder. Applying prospect the-

ory (with $\pi(p) = p$), Roberto will compare $\frac{4}{5}v(-200) + \frac{1}{5}v(800) : 0$ or $v(800) : 4(-v(-200))$. Now, diminishing sensitivity in the gain domain implies $v(800) < 4v(200)$, and loss aversion implies $v(200) < -v(-200)$. Putting these together, it follows that $v(800) < 4(-v(-200))$ and thus Roberto must choose the riskless option.

Question 5:

(a) The way to answer this question is to set up the inequalities implied by the choices, and to check whether both can hold simultaneously. Given Jennifer evaluates gambles according to prospect theory with $\pi(p) = p$, and using $v(0) = 0$, the first choice implies

$$\begin{aligned} .90v(800) &> .50v(1400) \\ \text{or } v(800) &> \frac{5}{9}v(1400). \end{aligned}$$

And the second choice implies

$$\begin{aligned} .25v(1400) &> .45v(800) \\ \text{or } \frac{5}{9}v(1400) &> v(800). \end{aligned}$$

Clearly the two inequalities cannot simultaneously hold, and so Jennifer could not exhibit this combination of choices.

Note that this combination of choices is an example of “subproportionality”, and the fact that Jennifer could not exhibit this combination of choices reveals that prospect theory does NOT accommodate subproportionality via the value function. However, prospect theory CAN explain this pattern via the probability-weighting function. In particular, when $\pi(p) \neq p$, the first choice implies

$$\begin{aligned} \pi(.90)v(800) &> \pi(.50)v(1400) \\ \text{or } v(800) &> \frac{\pi(.50)}{\pi(.90)}v(2500). \end{aligned}$$

And the second choice implies

$$\begin{aligned}\pi(.25)v(1400) &> \pi(.45)v(800) \\ \text{or } \frac{\pi(.25)}{\pi(.45)}v(1400) &> v(800).\end{aligned}$$

Hence, if $\frac{\pi(.25)}{\pi(.45)} > \frac{\pi(.50)}{\pi(.90)}$, then both inequalities can be satisfied. Recall that Kahneman & Tversky assume that the probability weighting function has exactly this feature.

(b) Here, Jennifer must choose between gamble $(-\$400, 1)$ and gamble $(-\$20,000, .02; \$0, .98)$. Applying the logic from question 3, because both gambles involve no possible gains, Jennifer will choose the risky gamble since it has the same expected value as the risky gamble (she is risk-loving in the loss domain). Hence, Jennifer would NOT purchase the insurance.

Under prospect theory with $\pi(p) \neq p$, there is a second effect. Now the person will purchase insurance when $v(-400) > \pi(0.02) * v(-20000)$. Because Kahneman & Tversky assume that small probabilities are overweighted — that is, $\pi(p) > p$ for small p — probability weighting will argue for purchasing the insurance. Hence, there are conflicting effects, and thus prospect theory does not make a prediction.

(c) Here, Jennifer must choose between gamble $(-\$1200, 1)$ and gamble $(-2000, .6; \$0, .4)$. Using exactly the same logic from part **(b)**, Jennifer would NOT purchase the insurance..

Again, under prospect theory with $\pi(p) \neq p$, there is the second effect, but now the second effect reinforces the decision NOT to insure. Specifically, because Kahneman & Tversky (implicitly) assume that $\pi(0.6) < 0.6$, probability weighting will argue for NOT insuring. Because the two effects both argue for NOT insuring, prospect theory unambiguously predicts that the person should NOT purchase insurance.

[Note that while Kahneman & Tversky are a little vague on exactly where π switches from $\pi(p) > p$ to $\pi(p) < p$, subcertainty implies that $\pi(0.5) < 0.5$, because it requires $\pi(0.5) + \pi(0.5) < 1$, and thus the switch must occur at some $p < 0.5$.]

Question 6:

(ai) Here, Heidi is choosing whether to accept the lottery (\$400, $\frac{1}{3}$; $-\$Y, \frac{2}{3}$). She rejects when

$$\frac{1}{3} \ln(12400) + \frac{2}{3} \ln(12000 - Y) < \ln(12000)$$

which we can use to solve for the critical Y :

$$\begin{aligned} \ln(12000 - Y) &< \frac{3}{2} \ln(12000) - \frac{1}{2} \ln(12400) \\ 12000 - Y &< \exp\left(\frac{3}{2} \ln(12000) - \frac{1}{2} \ln(12400)\right) \\ Y &> 195.135. \end{aligned}$$

(aii) If this bet is played twice, the aggregate bet is ($\$800, \frac{1}{9}$; $\$(400 - Y), \frac{4}{9}$; $-\$2Y, \frac{4}{9}$). Heidi accepts when

$$\frac{1}{9} \ln(12800) + \frac{4}{9} \ln(12400 - Y) + \frac{4}{9} \ln(12000 - 2Y) > \ln(12000)$$

which we can use to solve for the critical Y :

$$\begin{aligned} \ln(12400 - Y) + \ln(12000 - 2Y) &> \frac{9}{4} \ln(12000) - \frac{1}{4} \ln(12800) \\ (12400 - Y)(12000 - 2Y) &> \exp\left(\frac{9}{4} \ln(12000) - \frac{1}{4} \ln(12800)\right) \end{aligned}$$

or

$$2Y^2 - 36800Y + \left[(12400)(12000) - \exp\left(\frac{9}{4} \ln(12000) - \frac{1}{4} \ln(12800)\right)\right] > 0.$$

Applying the quadratic equation and using the lower root yields $Y < 195.133$.

(aii) NO, because it is not possible to have both $Y > 195.135$ and $Y < 195.133$.

(bi) Here, Bruce is choosing whether to accept the lottery (\$400, $\frac{1}{3}$; $-\$Y, \frac{2}{3}$). He rejects when

$$\begin{aligned}\frac{1}{3}v(400) + \frac{2}{3}v(-Y) &< 0 \\ \frac{1}{3}(400) + \frac{2}{3}(-2.5Y) &< 0 \\ Y &> 80.\end{aligned}$$

(bii) If this bet is played twice, the aggregate bet is ($\$800, \frac{1}{9}$; $\$(400 - Y), \frac{4}{9}$; $-\$2Y, \frac{4}{9}$). We have to be a little careful, because we must account for whether the critical Y is larger or smaller than 400, because that affects $v(400 - Y)$. Note that $Y = 400$ implies that Bruce evaluates the bet as $\frac{1}{9}v(800) + \frac{4}{9}v(-800) < 0$, and thus Bruce strictly prefers NOT to bet. It follows that the critical Y is smaller than 400. For $Y < 400$, Bruce accepts when

$$\begin{aligned}\frac{1}{9}v(800) + \frac{4}{9}v(400 - Y) + \frac{4}{9}v(-2Y) &> 0 \\ \frac{1}{9}(800) + \frac{4}{9}(400 - Y) + \frac{4}{9}(-5Y) &> 0 \\ Y &< 100\end{aligned}$$

(biii) YES, Bruce will reject the single bet but accept the aggregate bet if $Y \in (80, 100)$.

Question 7

(a) For a 50-50 bet of losing Y vs. gaining X , Johnny will accept when:

$$\frac{1}{2} \left(-e^{-0.001(w-Y)} \right) + \frac{1}{2} \left(-e^{-0.001(w+X)} \right) > -e^{-0.001w}.$$

Multiplying through by $-e^{0.001w}$ yields

$$\begin{aligned}\frac{1}{2} \left(e^{-0.001(-Y)} \right) + \frac{1}{2} \left(e^{-0.001(X)} \right) &< 1. \\ e^{0.001Y} + e^{-0.001(X)} &< 2 \\ -0.001X &< \ln [2 - e^{0.001Y}] \\ X &> \frac{-\ln [2 - e^{0.001Y}]}{0.001}.\end{aligned}$$

Note that this is independent of wealth, which is unique to this utility function. Plugging in the appropriate Y yields:

- Johnny will accept $(-100, \frac{1}{2}; X, \frac{1}{2})$ if and only if $X > \$111.12$.
- Johnny will accept $(-200, \frac{1}{2}; X, \frac{1}{2})$ if and only if $X > \$250.26$.
- Johnny will accept $(-500, \frac{1}{2}; X, \frac{1}{2})$ if and only if $X > \$1046.18$.
- Johnny will NOT accept $(-750, \frac{1}{2}; X, \frac{1}{2})$ for any X .

Note: For the last case, $e^{0.001Y} > 2$, and so, given that $e^{-0.001(X)} > 0$ for any X , it's impossible to ever have $e^{0.001Y} + e^{-0.001(X)} < 2$.

(b) Tommy will accept the lottery $(-100, \frac{1}{2}; X, \frac{1}{2})$ when

$$\begin{aligned}\frac{1}{2}v(-100) + \frac{1}{2}v(X) &> 0 \\ \frac{1}{2}(-\lambda 100) + \frac{1}{2}(X) &> 0 \quad \iff \quad X > \lambda 100.\end{aligned}$$

Since we're told that Tommy accepts if and only if $X > 210$, we can conclude that $210 = \lambda 100$, and so $\lambda = 2.1$.

(c) Using $\lambda = 2.1$, Tommy will accept a 50-50 bet of losing Y vs. gaining X when

$$\frac{1}{2}(-\lambda Y) + \frac{1}{2}(X) > 0 \quad \Longleftrightarrow \quad X > 2.1Y.$$

Hence:

- Tommy will accept $(-200, \frac{1}{2}; X, \frac{1}{2})$ if and only if $X > \$420$.
- Tommy will accept $(-500, \frac{1}{2}; X, \frac{1}{2})$ if and only if $X > \$1050$.
- Tommy will accept $(-750, \frac{1}{2}; X, \frac{1}{2})$ if and only if $X > \$1575$.

Question 8

(a) If you invest in all bonds, then over one month your return will be 0.8% with certainty. Given \$1000 to invest, you'll face lottery $x_B^1 = (\$8, 1)$.

If you invest in all stocks, then over one month your return will be 1.8% with probability 0.7 and -1.1% with probability 0.3. Given \$1000 to invest, you'll face lottery $x_S^1 = (\$18, 0.7; -\$11, 0.3)$.

Your prospective utility from the two options are:

$$E[v(x_B^1)] = v(8) = 8.$$

$$E[v(x_S^1)] = 0.7v(18) + 0.3v(-11) = 0.7(18) + 0.3(-24.75) = 5.175.$$

Hence, you prefer all bonds.

(b) If you invest in all bonds, then over two months your return will be $(1.008)(1.008) - 1 = 1.606\%$ with certainty. Given \$1000 to invest, you'll face lottery $x_B^2 = (\$16.06, 1)$.

If you invest in all stocks, then over two months your return will be $(1.018)(1.018) - 1 = 3.632\%$ with probability $0.7 * 0.7 = 0.49$, $(1.018)(0.989) - 1 = 0.680\%$ with probability $2 * 0.7 * 0.3 = 0.42$; and $(0.989)(0.989) - 1 = -2.188\%$ with probability $0.3 * 0.3 = 0.09$. Given \$1000 to invest, you'll face lottery $x_S^2 = (\$36.32, 0.49; \$6.80, 0.42; -\$21.88, 0.09)$.

Your prospective utility from the two options are:

$$E[v(x_B^2)] = v(16.06) = 16.06.$$

$$\begin{aligned} E[v(x_S^2)] &= 0.49v(36.32) + 0.42v(6.80) + 0.09v(-21.88) \\ &= 0.49(36.32) + 0.42(6.80) + 0.09(-49.23) = 16.22. \end{aligned}$$

Hence, you prefer all stocks.

(c) This question illustrates the basic intuition behind Benartzi & Thaler's explanation of the equity premium puzzle. If people choose their financial allocations based primarily on the gain-loss utility they experience each time they evaluate their portfolio, then if their portfolio evaluations are sufficiently frequent, they'll prefer bonds, whereas if their portfolio evaluations are sufficiently infrequent, they'll prefer stocks, and thus there is an evaluation horizon for which they are roughly indifferent between stocks and bonds. [In contrast, under expected utility, the evaluation horizon cannot matter in this way.]

(d) Using the returns from part (b), if you split your \$1000 evenly between stocks and bonds, the possible outcomes are:

If stocks do well both periods: $\$500(3.632\%) + \$500(1.606\%) = \$26.19$

If stocks do well in one period: $\$500(0.680\%) + \$500(1.606\%) = \$11.43$

If stocks do poorly in both periods: $\$500(-2.188\%) + \$500(1.606\%) = -\$2.91$

Hence, you'll face lottery $x_{SB}^2 = (\$26.19, 0.49 ; \$11.43, 0.42 ; -\$2.91, 0.09)$, and your prospective utility from this allocation is:

$$\begin{aligned} E[v(x_{SB}^2)] &= 0.49v(26.19) + 0.42v(11.43) + 0.09v(-2.91) \\ &= 0.49(26.19) + 0.42(11.43) + 0.09(-6.55) = 17.04. \end{aligned}$$

Hence, you prefer this mixed portfolio over a portfolio of all stocks (and over a portfolio of all bonds).

Question 9:

Let's first review the three types and the choices they face.

Buyers are unendowed and thus have mug reference point $r = 0$. For any given p , buyers can choose not to buy, in which case they have $c = 0$ and $m = w$, or they can choose to buy, in which case they have $c = 1$ and $m = w - p$.

Sellers are endowed and thus have mug reference point $r = 1$. For any given p , sellers can choose to keep the mug, in which case they have $c = 1$ and $m = w$, or they can choose to sell the mug, in which case they have $c = 0$ and $m = w + p$.

Choosers are unendowed and thus have mug reference point $r = 0$. For any given p , choosers can choose a mug, in which case they have $c = 1$ and $m = w$, or they can choose the money, in which case they have $c = 0$ and $m = w + p$.

(ai) For buyers, utility from buying is $(\mu + \eta_{\text{mug}}) + (w - p)$; the utility from not buying is $(0 + 0) + (w)$; and thus the reservation value is defined by $(\mu + \eta_{\text{mug}}) + (w - p) = w$. Hence, their reservation value is $P_B = \$(\mu + \eta_{\text{mug}})$.

(aii) For sellers, utility from keeping is $(\mu + 0) + (w)$; the utility from selling is $(0 + (-\lambda_{\text{mug}}\eta_{\text{mug}})) + (w + p)$; and thus the reservation value is defined by $\mu + w = (-\lambda_{\text{mug}}\eta_{\text{mug}}) + (w + p)$. Hence, their reservation value is $P_S = \$(\mu + \lambda_{\text{mug}}\eta_{\text{mug}})$.

(aiii) For choosers, utility from choosing mug is $(\mu + \eta_{\text{mug}}) + (w)$; the utility from choosing money is $(0 + 0) + (w + p)$; and thus the reservation value is defined by $(\mu + \eta_{\text{mug}}) + (w) = w + p$. Hence, their reservation value is $P_C = \$(\mu + \eta_{\text{mug}})$.

(aiv) As we discussed in class, when a person has linear money utility, loss aversion over mugs (i.e., $\lambda_{\text{mug}} > 1$) causes sellers to have a larger reservation value than both buyers and choosers, but there is no difference between buyers and choosers. E.g., for the special case when $\mu = 3$, $\eta_{\text{mug}} = 0.5$, and $\lambda_{\text{mug}} = 5$, the reservation values are $P_B = P_C = \$3.50$ and $P_S = \$5.50$.

(bi) For buyers, utility from buying is $(\mu + \eta_{\text{mug}}) + ((w - p) + (-\lambda_{\text{money}}\eta_{\text{money}}p))$; the utility from not buying is $(0 + 0) + (w + 0)$; and thus the reservation value is defined by $(\mu + \eta_{\text{mug}}) + w - (1 + \lambda_{\text{money}}\eta_{\text{money}})p = w$. Hence, their reservation value is $P_B = \$\left(\frac{\mu + \eta_{\text{mug}}}{1 + \lambda_{\text{money}}\eta_{\text{money}}}\right)$.

(bii) For sellers, utility from keeping is $(\mu + 0) + (w + 0)$; the utility from selling is $(0 + (-\lambda_{\text{mug}}\eta_{\text{mug}})) + ((w + p) + \eta_{\text{money}}p)$; and thus the reservation value is defined by $\mu + w = (-\lambda_{\text{mug}}\eta_{\text{mug}}) + w + (1 + \eta_{\text{money}})p$. Hence, their reservation value is $P_S = \$\left(\frac{\mu + \lambda_{\text{mug}}\eta_{\text{mug}}}{1 + \eta_{\text{money}}}\right)$.

(biii) For choosers, utility from choosing mug is $(\mu + \eta_{\text{mug}}) + (w + 0)$; the utility from choosing money is $(0 + 0) + ((w + p) + \eta_{\text{money}}p)$; and thus the reservation value is defined by $(\mu + \eta_{\text{mug}}) + w = w + (1 + \eta_{\text{money}})p$. Hence, their reservation value is $P_C = \$\left(\frac{\mu + \eta_{\text{mug}}}{1 + \eta_{\text{money}}}\right)$.

(biv) Here, loss aversion over mugs (i.e., $\lambda_{\text{mug}} > 1$) causes sellers to have a larger reservation value than choosers, exactly as above. But now loss aversion over money (i.e., $\lambda_{\text{money}} > 1$) causes choosers to have a larger reservation value than buyers — intuitively, because buyers are thinking about possibly losing money while choosers are thinking about possibly gaining money. E.g., for the special case when $\mu = 3$, $\eta_{\text{mug}} = 0.5$, $\lambda_{\text{mug}} = 5$, $\eta_{\text{money}} = 0.3$, and $\lambda_{\text{money}} = 4$, the reservation

values are $P_B = \$1.59$, $P_C = \$2.69$, and $P_S = \$4.23$.

(ci) For buyers, utility from buying is $(\mu + \eta_{\text{mug}}) + 15,000 \ln(w - p)$; the utility from not buying is $(0 + 0) + 15,000 \ln(w)$; and thus the reservation value is defined by $(\mu + \eta_{\text{mug}}) + 15,000 \ln(w - p) = 15,000 \ln(w)$. Using $w = 15,000$, $\mu = 3$, and $\eta_{\text{mug}} = 0.5$, we can derive that their reservation value is $P_B = \$3.4996$.

(cii) For sellers, utility from keeping is $(\mu + 0) + 15,000\ln(w)$; the utility from selling is $(0 + (-\lambda_{\text{mug}}\eta_{\text{mug}})) + 15,000\ln(w + p)$; and thus the reservation value is defined by $\mu + 15,000\ln(w) = -\lambda_{\text{mug}}\eta_{\text{mug}} + 15,000\ln(w + p)$. Using $w = 15,000$, $\mu = 3$, $\eta_{\text{mug}} = 0.5$, and $\lambda_{\text{mug}} = 5$, we can derive that their reservation value is $P_S = \$5.5010$.

(ciii) For choosers, utility from choosing mug is $(\mu + \eta_{\text{mug}}) + 15,000\ln(w)$; the utility from choosing money is $(0 + 0) + 15,000\ln(w + p)$; and thus the reservation value is defined by $(\mu + \eta_{\text{mug}}) + 15,000\ln(w) = 15,000\ln(w + p)$. Using $w = 15,000$, $\mu = 3$, and $\eta_{\text{mug}} = 0.5$, we can derive that their reservation value is $P_C = \$3.5004$.

(civ) Introducing diminishing marginal utility from money has the same qualitative effects as introducing loss aversion over money. Specifically, we still see the endowment effect — we still have $P_S > P_B$ and $P_S > P_C$ — but we now have a difference between buyers and choosers. Here, the intuition is that choosers are wealthier than buyers, and hence are willing to pay more for the mug — and so $P_C > P_B$. Note, however, the magnitude of the latter effect is very small.

(di) For buyers, utility from buying is $(3.5) + 15,000\ln(w - p)$; the utility from not buying is $(0) + 15,000\ln(w)$; and thus the reservation value is defined by $3.5 + 15,000\ln(w - p) = 15,000\ln(w)$. Using $w = 15,000$, we can derive that their reservation value is $P_B = \$3.4996$.

(dii) For sellers, utility from keeping is $(3.5) + 15,000\ln(w)$; the utility from selling is $(0) + 15,000\ln(w + p)$; and thus the reservation value is defined by $3.5 + 15,000\ln(w) = 15,000\ln(w + p)$. Using $w = 15,000$, we can derive that their reservation value is $P_S = \$3.5004$.

(diii) For choosers, utility from choosing mug is $(3.5) + 15,000\ln(w)$; the utility from choosing money is $(0) + 15,000\ln(w + p)$; and thus the reservation value is defined by $(3.5) + 15,000\ln(w) = 15,000\ln(w + p)$. Using $w = 15,000$, we can derive that their reservation value is $P_C = \$3.5004$.

(div) When we eliminate loss aversion over mugs, the endowment effect disappears. In particular, sellers and choosers have identical reservation values — $P_S = P_C$ — because they face exactly the same choice (they differ in only their mug reference point, which is now irrelevant). Also, because sellers and choosers are wealthier than buyers, buyers have a smaller reservation value. But note again that the magnitude of the latter effect is very small.

EXTRA QUESTIONS:

(a) In principle, NO the reflection effect does not violate expected utility. Suppose, for example, that we see a person choose $(1000, 1)$ over $(2000, \frac{1}{2}; 0, \frac{1}{2})$, but the same person also chooses $(-2000, \frac{1}{2}; 0, \frac{1}{2})$ over $(-1000, 1)$ — which is a typical reflection effect. Because these lotteries involve completely different sets of final wealth states, the combination does not violate expected utility. (But notice that the latter choice does violate expected utility with risk aversion.)

(b) Diminishing sensitivity applies to comparisons between different sized gains (or between different sized losses), and says that the sensitivity to gains (or losses) declines as those gains (or losses) get larger. In contrast, loss aversion applies to comparisons between gains and losses, and says that losses loom larger than equal sized gains.