Problem Set Lesley — Suggested Answers

Question 1:

(a) The expected values are:

$$
\begin{aligned}
EV(\mathbf{x}) &= 0.7(1000) + 0.3(-1000) = 400 \\
EV(\mathbf{y}) &= \frac{1}{3}(600) + \frac{1}{2}(300) + \frac{1}{6}(0) = 350.
\end{aligned}
$$

Hence, if Ting is an EV maximizer, she will choose lottery $\mathbf{x}$.

Note: You could instead calculate the expected final wealth from each lottery, and you'd reach the same conclusion: Ting will choose lottery $\mathbf{x}$. Unlike EU, under EV it doesn't matter whether one incorporates initial wealth.

(b) In general, the expected utilities are:

$$
\begin{aligned}
EU(\mathbf{x}) &= 0.7u(w+1000) + 0.3u(w-1000) \\
&= 0.7u(11,000) + 0.3u(9,000)
\end{aligned}
$$

$$
\begin{aligned}
EU(\mathbf{y}) &= \frac{1}{3}u(w+600) + \frac{1}{2}u(w+300) + \frac{1}{6}u(w) \\
&= \frac{1}{3}u(10,600) + \frac{1}{2}u(10,300) + \frac{1}{6}u(10,000).
\end{aligned}
$$

(bi) The CRRA utility function with $\rho = 1/2$ is $u(x) = 2x^{1/2}$, and thus the expected utilities become

$$
\begin{aligned}
EU(\mathbf{x}) &= 0.7 * 2(11,000)^{1/2} + 0.3 * 2(9,000)^{1/2} = 203.754 \\
\text{and} \quad EU(\mathbf{y}) &= \frac{1}{3} * 2(10,600)^{1/2} + \frac{1}{2} * 2(10,300)^{1/2} + \frac{1}{6} * 2(10,000)^{1/2} = 203.460.
\end{aligned}
$$

Hence, Grace will choose lottery $\mathbf{x}$.

(bii) The CRRA utility function with $\rho = 2$ is $u(x) = -x^{-1}$, and thus the expected utilities become

$$
\begin{aligned}
EU(\mathbf{x}) &= 0.7(-1/11{,}000) + 0.3(-1/9{,}000) = -9.697 * 10^{-5} \\
\text{and} \quad EU(\mathbf{y}) &= \frac{1}{3}(-1/10{,}600) + \frac{1}{2}(-1/10{,}300) + \frac{1}{6}(-1/10{,}000) = -9.666 * 10^{-5}.
\end{aligned}
$$

Hence, Sunflower will choose lottery **y**.

Question 2:

(a) If Sonya puts all of her wealth in the risk-free asset, she ends the year with $(20{,}000)(1.03) =$ $\$20{,}600$ for sure, so her expected utility is

$$EU(\text{risk-free}) = \ln(20{,}600) = 9.933.$$

If instead she puts all of her wealth in the risky asset, she faces the lottery

$$(\ \$29{,}000, 1/3\ ;\ \$17{,}000, 2/3\ ),$$

so her expected utility is

$$EU(\text{risky}) = \frac{1}{3}\ln(29{,}000) + \frac{2}{3}\ln(17{,}000) = 9.919.$$

Hence, Sonya will put all of her wealth in the risk-free asset.

Note: The risky asset has the higher expected value — $\frac{1}{3}(29{,}000) + \frac{2}{3}(17{,}000) = \$21{,}000$, against $20,600 — but Sonya is risk-averse, and putting everything in the risky asset is too much risk to be worth the extra expected value.

(b) To solve this part, we first solve for the lottery that Sonya will face. As in the problem, let y be the amount that she invests in the risky asset (although one could define the choice variable differently — e.g., the proportion that she invests in the risky asset — and one would reach the same conclusion).

If Sonya invests $\$y$ in the risky asset, she invests $\$(20,000-y)$ in the risk-free asset. Then if the risky asset does well, her final wealth will be

$$(y)(1.45)+(20,000-y)(1.03)=20,600+.42y.$$

If instead the risky asset does poorly, her final wealth will be

$$(y)(0.85)+(20,000-y)(1.03)=20,600-.18y.$$

Hence, as a function of y, the lottery she faces is

$$(\ 20,600+.42y, 1/3\ ;\ 20,600-.18y, 2/3\ ),$$

and therefore, as a function of y, her expected utility is

$$EU(y)=\frac{1}{3}\ln(20,600+.42y)+\frac{2}{3}\ln(20,600-.18y).$$

Sonya will then choose y to maximize $EU(y)$. The first-order condition is

$$\begin{aligned}
0 &= \frac{dEU(y)}{dy} \\
0 &= \frac{1}{3}*\left(\frac{.42}{20,600+.42y}\right) \quad + \quad \frac{2}{3}*\left(\frac{-.18}{20,600-.18y}\right) \\
\frac{2}{3}*\left(\frac{.18}{20,600-.18y}\right) &= \frac{1}{3}*\left(\frac{.42}{20,600+.42y}\right) \\
0.12*(20,600+.42y) &= 0.14*(20,600-.18y) \\
y &= \$5449.74.
\end{aligned}$$

This lies between \$0 and \$20,000, so Sonya will invest \$5449.74 in the risky asset, and thus $\$14,550.26$ in the risk-free asset.

Note: Part (a) might suggest that Sonya wants nothing to do with the risky asset, yet she puts some of her wealth in it here. Because the risky asset has the higher expected return, a small investment in it raises her expected wealth while adding very little risk, so she keeps investing until the added risk begins to outweigh the extra return.

Question 3:

(a) Because Josh is a risk-averse expected utility maximizer, he has two concerns: he dislikes risk, and he likes expected value. If these two concerns argue for the same lottery, then we can say what Josh will do — choose that lottery. But if these two concerns argue for different lotteries, then we need additional information — because we'd need to know which concern is stronger. Hence:

For choice (i), the expected value of the risky lottery is \$1250, and because this is smaller than the sure amount of \$1300, liking expected value argues for choosing the riskless lottery. Disliking risk of course also argues for the riskless lottery. Because the two concerns agree, we can conclude that Josh will choose the riskless (the second) lottery.

For choice (ii), the expected value of the risky lottery is $-\$2250$, and because this is larger than the sure amount of $-\$2300$, liking expected value argues for choosing the risky lottery. At the same time, disliking risk of course argues for the riskless lottery. Because the two concerns conflict, we need more information to determine which option Josh will choose.

For choice (iii), the expected value of both lotteries is \$300, and thus liking expected value says that either is fine. The key thing to note here is that the first lottery is riskier than the second lottery in the sense that it is a mean-preserving spread — in particular, some of the probability from \$400 and \$200 is put instead on \$600 and \$0. Hence, disliking risk argues for the second lottery. Because liking expected value is indifferent while disliking risk argues for the second lottery, we can conclude that Josh will choose the second lottery.

(b) For choices (i) and (iii), we already know that Josh will choose the second option no matter his risk aversion. For choice (ii), choosing the risky (the first) lottery involves getting a higher expected value but taking on more risk. As Josh becomes more risk-averse, he is less willing to do this — that is, as he becomes more risk-averse, he becomes more prone to choose the second option.

(c) Recall that a CRRA utility function implies that risk aversion decreases as Josh's wealth goes up. Applying our conclusions from part (b), for choices (i) and (iii), Josh chooses the second option no matter his wealth, whereas for choice (ii), as his wealth goes up, Josh becomes more prone to choose the first option.

Question 4:

(a) If you do not buy any insurance, you face lottery

$$(\ \$10,000, 1\%\ ;\ \$18,500, 2\%\ ;\ \$19,500, 8\%\ ;\ \$19,900, 20\%\ ;\ \$20,000, 69\%).$$

If you buy the insurance, each of the payments is reduced by 90%, but you also must pay the premium p no matter what happens. Hence, you face lottery

$$(\ \$19,000-p, 1\%\ ;\ \$19,850-p, 2\%\ ;\ \$19,950-p, 8\%\ ;\ \$19,990-p, 20\%\ ;\ \$20,000-p, 69\%).$$

(b) At price p^*, you are indifferent between these two lotteries — that is,

$$(\ \$10,000, 1\%\ ;\ \$18,500, 2\%\ ;\ \$19,500, 8\%\ ;\ \$19,900, 20\%\ ;\ \$20,000, 69\%) \sim$$

$$(\ \$19,000-p^*, 1\%\ ;\ \$19,850-p^*, 2\%\ ;\ \$19,950-p^*, 8\%\ ;\ \$19,990-p^*, 20\%\ ;\ \$20,000-p^*, 69\%).$$

For an expected utility maximizer, this indifference condition implies

$$.01u(10,000)+.02u(18,500)+.08u(19,500)+.20u(19,900)+.69u(20,000) =$$

$$.01u(19,000-p^*)+.02u(19,850-p^*)+.08u(19,950-p^*)+.20u(19,990-p^*)+.69u(20,000-p^*).$$

(c) If you are risk-neutral, your willingness to pay for the insurance p^* will be exactly equal to the expected payout that you expect to receive from the insurance company. Here, the expected payout is

$$.01(\$9000) + .02(\$1350) + .08(\$450) + .20(\$90) = \$171.$$

Hence, if you are risk-neutral, then your $p^* = \$171$.

If you are risk-averse, because the insurance reduces risk, you will be willing to pay more than the actuarially fair price for the insurance — that is, your $p^* > \$171$. How much greater, we cannot say without more information.

Finally, as you become more risk averse, you become more willing to pay a fee to reduce risk. Hence, your p^* will increase.

(d) If the coinsurance rate goes up to 15%, this means that you are receiving less insurance — you are now covered for only 85% of your medical payments instead of 90%. Hence, you'll be willing to pay less for this insurance — i.e., your p^* will decrease.

Question 5:

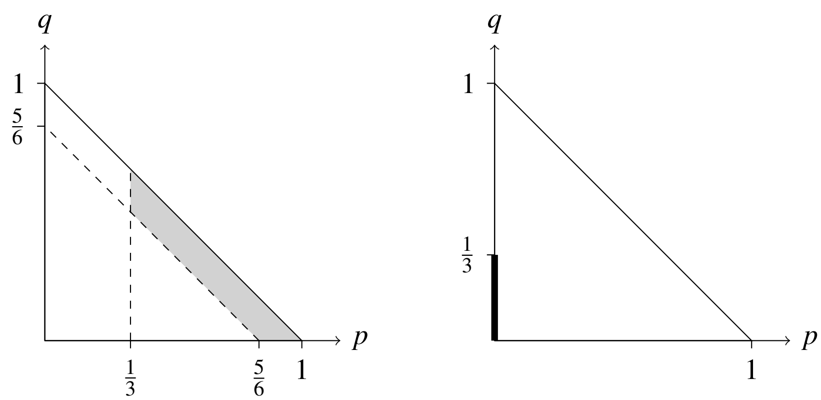
The direct way to answer this question is to develop a table for the likelihood of getting a payoff at least as large as z for various values of z:

	$\Pr(\text{payoff} \geq z)$	
	under Lottery **x**	under Lottery **y**
For any $z \leq 0$	1	1
For any $0 < z \leq 300$	$p+q$	5/6
For any $300 < z \leq 400$	$p+q$	1/3
For any $400 < z \leq 500$	p	1/3
For any $500 < z \leq 600$	p	0
For any $z > 600$	0	0

(a) For **x** to dominate **y**, it must be that the entry in the left column is larger than the entry on the right column in every row — weakly larger in every row, strictly larger in some row. Inspection of the table reveals that this condition will hold if and only if (i) $p \geq 1/3$ and (ii) $p+q \geq 5/6$. The combination of these two conditions is depicted graphically in the left-hand diagram below.

(b) For $\mathbf{y}$ to dominate $\mathbf{x}$, it must be that the entry in the right column is larger than the entry on the left column in every row — weakly larger in every row, strictly larger in some row. Inspection of the table reveals that this condition will hold if and only if (i) $p = 0$ and (ii) $q \leq 1/3$. The combination of these two conditions is depicted graphically in the right-hand diagram below.

(a) **x** dominates **y** (b) **y** dominates **x**



Question 6:

(a) We CANNOT say. In principle, a risk-averse expected utility maximizer could exhibit risk aversion that either increases or decreases with wealth (mathematically, some functional forms for u imply that risk aversion increases with wealth, while other functional forms imply that risk aversion decreases with wealth). However, economists typically make the further ASSUMPTION that risk aversion declines with wealth, because that is what we typically see in the data.

(b) NO, we can be certain that Julian would choose $(1000, \frac{1}{2}; 1100, \frac{1}{2})$ over $(1000, 1)$. This is because whenever we say that someone is a "risk-averse expected utility maximizer", we are implicitly assuming that the person also obeys more is better. And since $(1000, \frac{1}{2}; 1100, \frac{1}{2})$ dominates $(1000, 1)$, we know that Julian will choose $(1000, \frac{1}{2}; 1100, \frac{1}{2})$.

(c) NO, the St. Petersburg Paradox suggests that EXPECTED VALUE (and not expected utility) is a bad model of human behavior. Indeed, it is the aversion to risk suggested by the St. Petersburg Paradox that led economists to propose expected utility as a better model of human behavior — because it can capture that people both (i) like expected value and (ii) dislike risk.