Lecture 4B: Non-Bayesian Information Processing

EC 404: Behavioral Economics
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An Alternative Model: The Law of Small Numbers

An Alternative Model: The Law of Small Numbers
Based on Rabin (*QJE* 2002)

Assumes one particular misconception of randomness: People act as if they believe recent draws of one outcome increase the odds of the next draw being a different outcome.

Specifically, an *N-Freddy* acts as if he believes that probabilistic events are drawn from an "urn" with N balls without replacement — specifically, he believes that the ball just drawn is not in the "urn" for the current draw.

An Example

Suppose ($N = 12$)-Freddy faces a die with 4 H's and 2 M's.

On the first roll: $\Pr(H) = 2/3$

On the second roll:

- If first roll is H, then $\Pr(H) = 7/11 < 2/3$
- If first roll is M, then $\Pr(H) = 8/11 > 2/3$

More generally:

- If previous roll is H, then $\Pr(H) =$?
- If previous roll is M, then $\Pr(H) =$?

Formal Assumptions

Suppose the objective probability of outcome x is p.

On the first draw:

- N-Freddy believes $\Pr(x) = p$.

On the second draw:

- If first draw is x, N-Freddy believes $\Pr(x) = \frac{pN-1}{N-1}$.
- If first draw NOT x, N-Freddy believes $\Pr(x) = \frac{pN}{N-1}$.

More generally, on n^{th} draw (where $n \geq 2$):

- If $(n-1)^{\text{st}}$ draw is x, N-Freddy believes $\Pr(x) = \frac{pN-1}{N-1}$.
- If $(n-1)^{\text{st}}$ draw NOT x, N-Freddy believes $\Pr(x) = \frac{pN}{N-1}$.

A nice feature: The larger is N, the smaller is the bias
(and as $N \to \infty$ an N-Freddy becomes a Bayesian).

Implication 1

Implication 1: An N-Freddy will exhibit the gambler's fallacy.

- If there is a known objective probability, then an N-Freddy will act as if he believes a recent draw of one outcome increases the odds of the next draw being a different outcome (nearly tautological).

Implication 2

Implication 2: An N-Freddy will exhibit over-inference from small samples.

Recall Example C: Suppose there are two dice, a red one and a blue one. On the red die, there are 4 H's and 2 M's, whereas on the blue die, there are 2 H's and 4 M's. I flip a fair coin, and if it comes up heads I use the red die, and if it comes up tails I use the blue die (and you cannot observe the coin flip). I then roll that die twice. If the die comes up HH, what is the likelihood that I'm rolling the red die?

For a Bayesian: $q_0(\text{red}) = q_0(\text{blue}) = 1/2.$

$\gamma(HH|\text{red}) = 4/9$ and $\gamma(HH|\text{blue}) = 1/9.$

$$q_1(\text{red}|HH) = \frac{\gamma(HH|\text{red})q_0(\text{red})}{\gamma(HH|\text{red})q_0(\text{red}) + \gamma(HH|\text{blue})q_0(\text{blue})} = 4/5.$$

Implication 2

For an $(N = 12)$-Freddy:

- $q_0(\text{red}) = q_0(\text{blue}) = 1/2$ (no change).
- $\gamma(HH|\text{red}) = 2/3 * 7/11 = 14/33$
- $\gamma(HH|\text{blue}) = 1/3 * 3/11 = 1/11$
- Now apply Bayes' Rule:

$$q_1(\text{red}|HH) = \frac{\gamma(HH|\text{red})q_0(\text{red})}{\gamma(HH|\text{red})q_0(\text{red}) + \gamma(HH|\text{blue})q_0(\text{blue})} = \qquad 14/17$$

Intuition: The law of small numbers makes Freddy think that *HH* is less likely to occur than it actually is; but since this reduction is larger for the die less likely to roll *HH* (i.e., for the blue die), Freddy will update too much toward the die more likely to roll *HH* (i.e., toward the red die).

Implication 3

Implication 3: An N-Freddy will exhibit the hot-hand fallacy.

Implication 3

A Simplified Example:

- Suppose that in each game a player takes two shots. In reality, the player's hit rate is always 50%.

Consider an ($N = 8$)-Freddy who entertains the notion that the player might have hot and cold games. Specifically, for any given game, he believes:

- With probability q, the player's hit rate is 75%.
- With probability q, the player's hit rate is 25%.
- With probability $1 - 2q$, the player's hit rate is 50%.

Initially, Freddy doesn't know q, but Freddy attempts to infer q from watching the player's performance across a large number of games. What will Freddy infer?

Note: Let's assume that Freddy's processing reflects that there is a new urn at the start of every game.

Implication 3

What does Freddy actually see?

Given that the true hit rate is always 50%, after observing a "large" sample, Freddy will see pairs in the following proportions:

- $(H, H) = 1/4$
- $(M, M) = 1/4$
- (H, M) or $(M, H) = 1/2$

Implication 3

What does Freddy expect to see?

Recall that Freddy believes:

- With probability q, the player's hit rate is 75%.
- With probability q, the player's hit rate is 25%.
- With probability $1-2q$, the player's hit rate is 50%.

Hence, as a function of q, our $(N=8)$-Freddy expects to see pairs in the following proportions:

- $(H, H) = 3/14 + q/7$
- $(M, M) = 3/14 + q/7$
- (H, M) or $(M, H) = 8/14 - 2q/7$

Implication 3

What does Freddy infer about q?

Recall: Freddy actually sees:

- (H, H) — $1/4$
- (M, M) — $1/4$
- (H, M) or (M, H) — $1/2$

Recall: Freddy expects to see:

- (H, H) — $3/14 + q/7$
- (M, M) — $3/14 + q/7$
- (H, M) or (M, H) — $8/14 - 2q/7$

Hence, an $N = 8$ Freddy's estimate q will be the q such that . . .

Implication 3

Even though the player does NOT have a hot hand, Freddy will come to believe that the player is "hot" in 25% of games, "cold" in 25% of games, and "average" in 50% of games.

Intuition: The law of small numbers makes Freddy think that a 50% shooter is less likely to have *HH* or *MM* games than the player actually is. Hence, when Freddy sees a lot of *HH* and *MM* games, Freddy is forced to conclude that the player must be having hot and cold games.